Optimal Sovereign Defaults in the Presence of Financial Frictions

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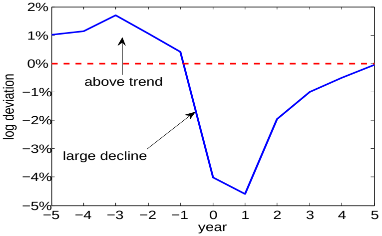
Introduction

- Sovereign defaults are often accompanied by large declines in output.
- Questions:
  1. Why are there declines in real economic activity?
  2. What determines the government's default decision?
- In this paper:
  1. Following default, financial disruptions lead to fall in output.
  2. Default is: beneficial due to reduced tax distortions;
     costly due to output declines.

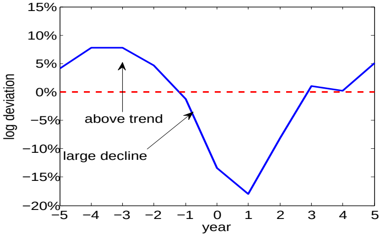
Motivating Evidence on Default and Economic Activity

- Event study analysis of 23 default episodes.
- Real economic activity is typically above trend before default.
- On average,
  1. output falls about 5%;
  2. investment falls about 17%;
  3. consumption falls about 3%;
  4. employment falls about 2%.

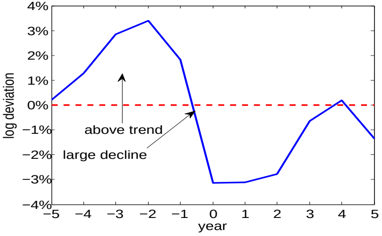
Macroeconomic Dynamics around Default Episodes



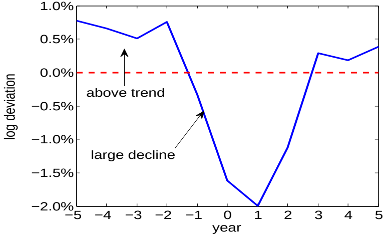
(a) Output



(b) Investment



(c) Consumption



(d) Employment

Key Mechanism for Output Decline

- Add financial frictions to a standard business cycle model.
  - Working capital must be borrowed and collateralized.
  - Physical capital and government bonds can be used as collateral.
- Default reduces collateral, and therefore working capital.
- Declines in working capital lead to fall in output.

Key Mechanism for Default Decisions

- Default gains are large when TFP is low.
  - Has to levy higher tax rate to repay debt than in normal times.
  - Reduces distortions by larger amount.
- Default losses are small when capital stock is high.
  - Higher capital stock implies higher collateral level.
  - Higher collateral implies lower financial frictions.
- Defaults typically occur after
  - a sequence of positive shocks and then,
  - a large negative shock.

Quantitative Findings

- In the data:
  - Argentina: more volatile TFP, more defaults.
  - Italy: less volatile TFP, no defaults.
- In the model: more volatile TFP leads to more frequent default.
  - Argentina: lower debt with relatively high default rate.
  - Italy: higher debt with negligible default rate.

Related Literature

- Default with Endowment Economy
  - Aguiar and Gopinath (2006), Arellano (2007), Chatterjee and Eyigungor (2012)
- Default with Production Economy
  - Mendoza and Yue (2012), Sosa-Padilla (2014), Bocola (2015), Perez (2015)
- Financial Frictions
  - Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), Jermann and Quadrini (2012)

Roadmap

- Model
- Equilibrium
- Quantitative analysis
- Conclusion

Environment

- Time: infinite horizon, discrete
- Agents:
  - measure 1 of identical households
  - measure 1 of identical firms
  - measure 1 of identical banks
  - government

Preferences and Technology

- Households' preferences:

$$U(c, n) = \log(c) - \chi \frac{n^{1+\nu}}{1+\nu}$$

- Firms' production technology:

$$F(z, k, n) = zk^{\alpha} n^{1-\alpha}$$

- Aggregate states: $S = (z, K, B)$
  - z exogenous
  - K and B endogenous
  - States evolve according to $S' = \Gamma(S)$

Households' Problem

- Taking policy, price and dividend functions as given,

$$W^h(e; S) = \max_{c,n,e'} U(c, n) + \beta E \left[W^h(e'; S') \right]$$

$$\text{s.t. } c + p(S)e' = [1 - \tau(S)]w(S)n + [p(S) + d(S)]e + \pi(S)$$

where

e :	household's holding of equity in bank
$w(S)$:	wage function
$p(S)$:	equity price function
$d(S)$:	bank's dividend function
$\tau(S)$:	tax function
$\pi(S)$:	firm's profit function

Firms' Problem

- Firms rent capital k from banks and hire labor n from households.
- Firms take intra-period working capital loans ℓ^f from banks.
- Working capital loan is to guarantee payments for capital and labor.
- Taking price functions as given, a firm's problem is

$$\begin{aligned} \pi(S) &= \max_{k,n,\ell^f} F(z,k,n) - r(S)k - w(S)n + \ell^f - \ell^f \\ \text{s.t. } \ell^f &\geq r(S)k + w(S)n \end{aligned}$$

Banks: Budget Constraint

- Banks start of period with capital k and bonds b.
- Banks take intra-period deposits ℓ^h from households.
- Banks make working capital loan ℓ^f to firms.
- A bank's budget constraint is

$$d + k' + q(S)b' = (1-\delta)k + r(S)k + b + (\ell^h - \ell^h) + (\ell^f - \ell^f)$$

where

$$\begin{aligned} d: &\quad \text{dividend} \\ q(S): &\quad \text{price of bonds} \end{aligned}$$

Banks: Collateral Constraint

- Banks can choose not to repay households.
- Households can only recover fraction ξ of net worth $k' + qb'$.
- The collateral constraint is

$$\xi(k' + qb') \geq \ell^h$$

Game Details

Banks' Problem

- Taking policy and price functions as given,

$$
\begin{aligned}
W^b(k, b; S) &= \max_{d,k',b',\ell^h,\ell^f} d + \beta E\left[\frac{U_c(c', n')}{U_c(c, n)} W^b(k', b'; S')\right] \\
\text{s.t.} & \\
d + k' + q(S)b' &= (1-\delta)k + r(S)k + [1 - D(S)\lambda]b \\
&\quad +(\ell^h - \ell^h) + (\ell^f - \ell^f) \\
\ell^h &= \ell^f \\
\xi[k' + q(S)b'] &\geq \ell^h
\end{aligned}
$$

where

$D(S)$: government default policy

λ : haircut on debt

Government

- Policy instruments:
  1. levy proportional taxes on labor;
  2. issue new debt;
  3. possibly default on outstanding debt.
- Finances exogenous public spending.
- Maximizes households' welfare.
- Has no commitment to its policies.

Government Budget Constraint

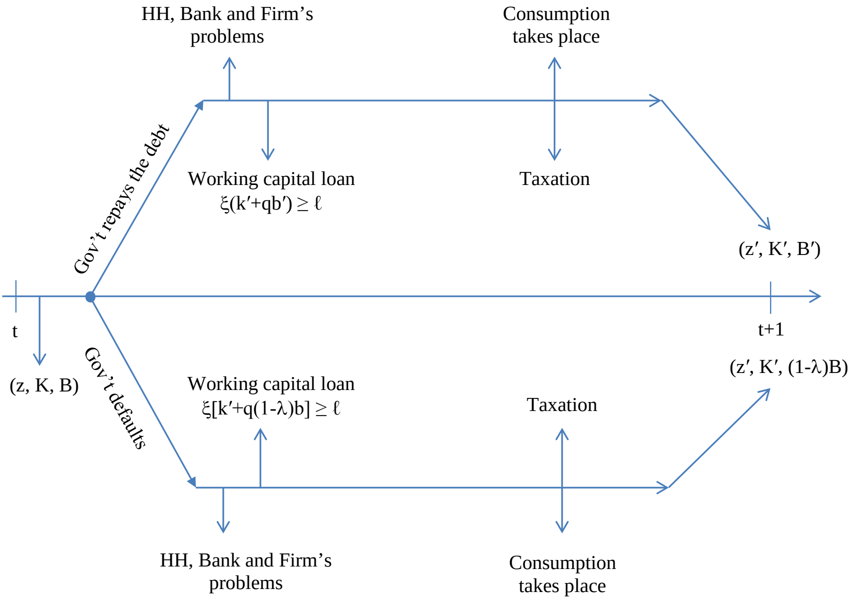
- If repays,

$$gY(S) + B = q(S)B'(S) + \tau(S)w(S)n(S)$$

- If defaults,
  - writes off fraction λ of the debt, and cannot issue new debt.
  - gets back to the credit market next period.

$$gY(S) + (1-\lambda)B = q(S)(1-\lambda)B + \tau(S)w(S)n(S)$$

Timing of Events



Government's Decisions to Default

- At the beginning of the period, government decides if it will default.

$$
\begin{aligned}
V(z,k,b) &= \max\left\{V^r(z,k,b), V^d(z,k,b)\right\} \\
D=0 \quad \text{if} &\quad V^r(z,k,b) \geq V^d(z,k,b) \\
D=1 \quad \text{if} &\quad V^r(z,k,b) < V^d(z,k,b)
\end{aligned}
$$

where

$$
\begin{aligned}
V^r: &\quad \text{value of repaying} \\
V^d: &\quad \text{value of defaulting}
\end{aligned}
$$

Government's Problem - Repay

$$
\begin{aligned}
V^r(z,k,b) &= \max_{c,n,d,k',b',\tau,w,q,\mu} U(c,n) + \beta E\left[V(z',k',b')\right] \\
\text{subject to} & \\
gzk^{\alpha}n^{1-\alpha} + b &= qb' + \tau wn \\
c &= (1-\tau)wn + d \\
d + k' + qb' &= (1-\delta)k + zk^{\alpha}n^{1-\alpha} - wn + b \\
\frac{U_n}{U_c} &= -(1-\tau)w \\
(1-\alpha)zk^{\alpha}n^{-\alpha} &= \frac{w}{1-\mu} \\
(1-\xi\mu)q &= \beta E\left(\frac{U_c(S')}{U_c}\left[1 - D(S')\lambda\right]\right) \\
1-\xi\mu &= \beta E\left(\frac{U_c(S')}{U_c}\left[1-\delta+(1-\mu(S'))\alpha z' k'^{\alpha-1} n(S')^{1-\alpha}\right]\right) \\
\xi(k'+qb') &\geq zk^{\alpha}n^{1-\alpha}, \mu \geq 0, \text{ and } \mu[\xi(k'+qb') - zk^{\alpha}n^{1-\alpha}] = 0
\end{aligned}
$$

Government's Problem - Default

$$
\begin{aligned}
V^d(z,k,b) &= \max_{c,n,d,k',\tau,w,q,\mu} U(c,n) + \beta E\left[V(z',k',(1-\lambda)b)\right] \\
&\text{subject to} \\
gzk^{\alpha}n^{1-\alpha} + (1-\lambda)b &= q(1-\lambda)b + \tau wn \\
c &= (1-\tau)wn + d \\
d + k' + q(1-\lambda)b &= (1-\delta)k + zk^{\alpha}n^{1-\alpha} - wn + (1-\lambda)b \\
\frac{U_n}{U_c} &= -(1-\tau)w \\
(1-\alpha)zk^{\alpha}n^{-\alpha} &= \frac{w}{1-\mu} \\
(1-\xi\mu)q &= \beta E\left(\frac{U_c(S')}{U_c}\left[1 - D(S')\lambda\right]\right) \\
1-\xi\mu &= \beta E\left(\frac{U_c(S')}{U_c}\left[1-\delta+(1-\mu(S'))\alpha z' k'^{\alpha-1} n(S')^{1-\alpha}\right]\right) \\
\xi(k'+q(1-\lambda)b) &\geq zk^{\alpha}n^{1-\alpha}, \mu \geq 0, \text{ and } \mu[\xi(k'+q(1-\lambda)b) - zk^{\alpha}n^{1-\alpha}] = 0
\end{aligned}
$$

Markov-Perfect Equilibrium

A Markov-Perfect Equilibrium is a set of value functions and policy functions for government, price functions, and allocation functions such that:

1. given price functions, allocation functions and future government policy functions, current government policy functions solve the government's problem;

2. given price functions and government policy functions, allocation functions are consistent with competitive equilibrium;

3. policy functions obtained by solving government problem coincide with future government policy functions that government problem takes as given;

4. markets clear

Tradeoffs in Government Policies

- Optimal for government to make labor wedge as small as possible.

- Labor wedge obtained from labor supply and demand equations

$$
\begin{aligned}
-\frac{U_n(c,n)}{U_c(c,n)} &= (1-\tau)w \\
F_n(z,k,n) &= \frac{w}{1-\mu}
\end{aligned}
$$

where μ is Lagrange multiplier on the collateral constraint.

- Combine to get

$$
-\frac{U_n(c,n)}{U_c(c,n)} = \underbrace{(1-\tau)(1-\mu)}_{\text{Labor Wedge}} F_n(z,k,n)
$$

- Default decreases taxes (τ) but increases financial frictions (μ).

Quantitative Analysis

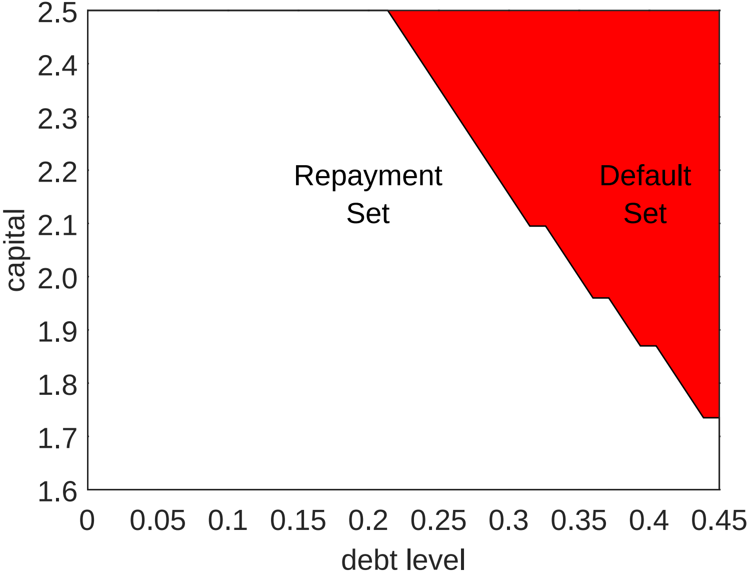
Quantitative Exercise

- Study Argentina and Italy.
- Use standard parameters and national accounts data.
- Estimate an AR(1) TFP process from data.
- Test if model can generate:
  - default frequencies
  - default patterns
  - output and investment declines

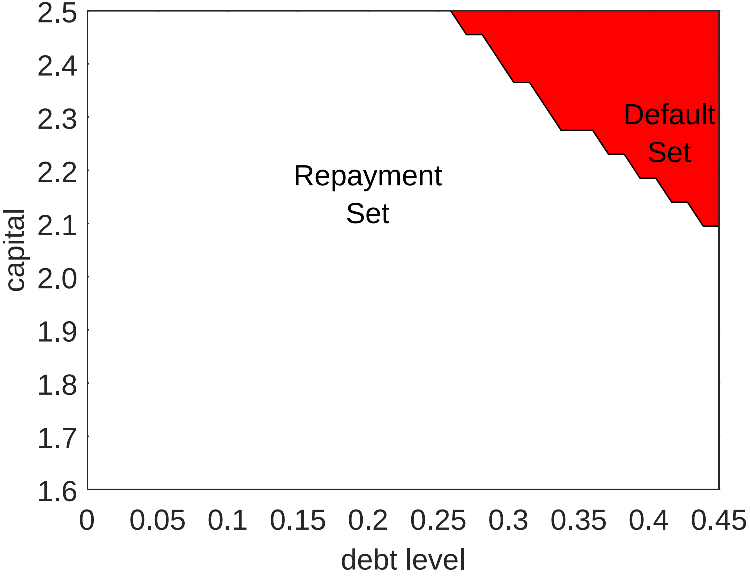
Parameters: Argentine data, annual frequency

Calibrated Parameters		Value	Target Statistics
Discount factor	β	0.95	Standard value
Disutility of labor	χ	4.18	Steady state hours $= 0.33$
Curvature of labor supply	ν	0.5	Frisch elasticity $= 2$
Capital share in output	α	0.3	Standard value
Capital depreciation rate	δ	0.1	Investment/GDP $= 20\%$
Govt spending/GDP	g	0.23	Govt spending/GDP $= 23\%$
Partial default	λ	0.55	Haircut $= 55\%$
Collateral parameter	ξ	0.440	Mean debt/GDP $= 27\%$
Autocorr. of prod shock	ρ_z	0.813	Autocorr. of TFP $= 0.813$
Std. dev. of prod shock	σ_z	0.046	Std. dev. of TFP $= 0.046$

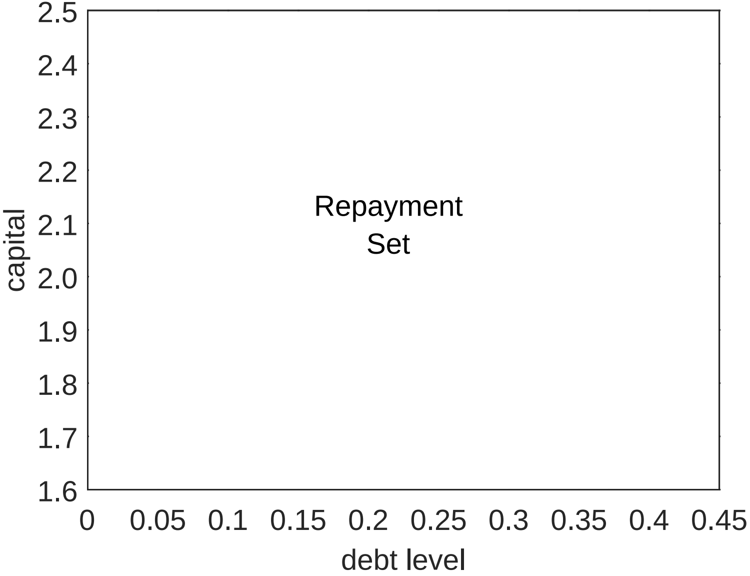
Default and Repayment Set for Low Productivity Shock



Default and Repayment Set for Mean Productivity Shock



Default and Repayment Set for High Productivity Shock

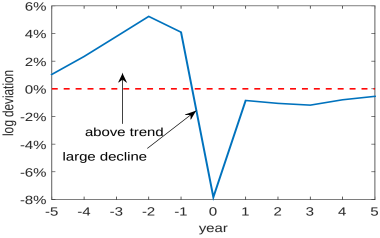


Argentina: Simulations

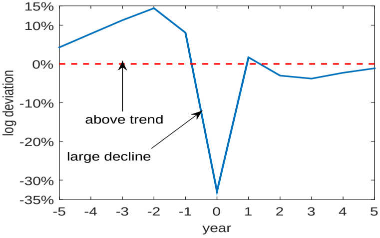
	Data	Model
Default probability	0.75%	0.78%
Mean output drop	11%	11.6%
Mean investment drop	36%	33.0%
Correlation btw default and GDP	−0.11	−0.127
Fraction of defaults with large recessions	32%	32.1%
Fraction of defaults with GDP below trend	62%	91.4%

Business Cycle Statistics

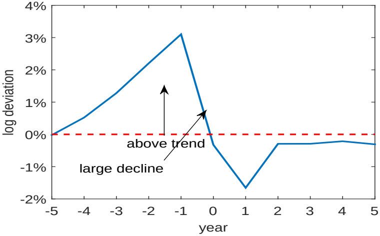
Argentina: Macro Dynamics around Default



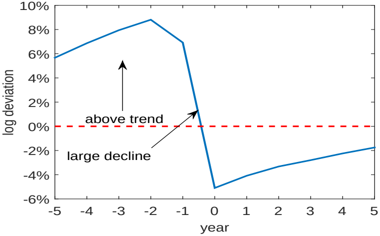
(a) Output



(b) Investment



(c) Consumption



(d) TFP

Argentina: Sensitivity Analysis

	Default probability	Output drop	Investment drop	$\sigma(r^s)$	$corr(y, r^s)$
Data	0.75%	11%	36%	2.51	-0.62
Baseline	0.78%	11.6%	33.0%	1.21	-0.64
Partial default rate (baseline: $\lambda = 0.55$)					
$\lambda = 0.5$	1.15%	10.7%	33.1%	1.54	-0.56
$\lambda = 0.6$	0.57%	12.1%	33.6%	1.22	-0.68
$\lambda = 0.7$	0.28%	12.9%	34.3%	1.17	-0.74
Frisch elasticity (baseline: $\frac{1}{\nu} = 2$)					
$\frac{1}{\nu} = 3$	0.45%	12.6%	35.8%	0.87	-0.66
$\frac{1}{\nu} = 1.5$	0.95%	11.1%	31.8%	1.30	-0.68
Enforcement constraint (baseline: $\xi = 0.44$)					
$\xi = 0.43$	0.63%	11.6%	31.6%	1.34	-0.75
$\xi = 0.45$	0.94%	10.8%	32.1%	1.01	-0.52

Parameters: Italian data, annual frequency

Calibrated Parameters		Value	Target Statistics
Discount factor	β	0.95	Standard value
Disutility of labor	χ	4.27	Steady state hours = 0.32
Curvature of labor supply	ν	0.5	Frisch elasticity = 2
Capital share in output	α	0.3	Labor income share = 0.7
Capital depreciation rate	δ	0.1	Investment/GDP = 20%
Govt spending/GDP	g	0.21	Govt spending/GDP = 21%
Partial default	λ	0.55	Haircut = 55%
Collateral parameter	ξ	0.390	Mean debt/GDP = 59%
Autocorr. of prod shock	ρ_z	0.925	Autocorr. of TFP = 0.925
Std. dev. of prod shock	σ_z	0.020	Std. dev. of TFP = 0.020

Italy: Simulations

- Use same preferences as for Argentina.
- Change TFP process and collateral constraint parameter.
- Get default rate: 0.028% in the model (0% in the data).
- If defaults, output decreases around 6.0%.

Counterfactual Analysis on Argentina

	Default probability	Output drop	Investment drop	$\sigma(r^s)$	$corr(y, r^s)$
Data	0.75%	11%	36%	2.51	-0.62
Baseline	0.78%	11.6%	33.0%	1.21	-0.64
Argentina has Italy's financial friction: $\xi = 0.39$					
	0.27%	11.8%	26.2%	2.05	-0.79
Argentina has Italy's productivity process: $\rho_z = 0.925, \sigma_z = 0.020$					
	0.24%	4.6%	5.8%	0.57	-0.50

Testable Implication

- Testable implication:
  default risk is higher when capital stock is higher.

- Panel regression:

$$S_{it} = \alpha_i + \delta_t + \beta K_{it} + \gamma X_{it} + \epsilon_{it}$$

where

$$\begin{aligned} S: &\quad \text{sovereign spread from EMBI;} \\ K: &\quad \text{capital to GDP ratio;} \\ X: &\quad \text{debt to GDP ratio;} \\ &\quad \text{real GDP growth rate;} \\ &\quad \text{current account to GDP ratio;} \\ &\quad \text{inflation.} \end{aligned}$$

Capital Stock and Sovereign Spread

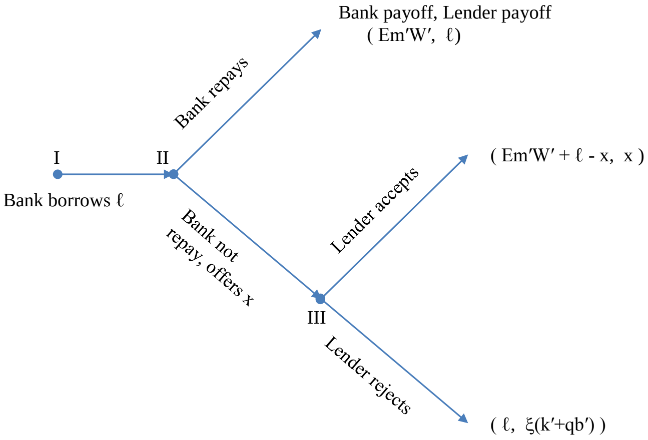
	Panel FE (1)	Panel FE (2)	Panel FE (3)	Panel RE (4)
Capital/GDP	2.10***	0.76*	0.74*	0.95***
	(0.33)	(0.42)	(0.37)	(0.28)
Debt/GDP		0.31***	0.31***	0.31***
		(0.09)	(0.09)	(0.09)
GDP growth rate			0.05	0.01
			(0.13)	(0.12)
Current account/GDP			0.08	0.20
			(0.12)	(0.13)
Inflation			0.0056***	0.0048***
			(0.0009)	(0.0010)

Conclusions

- Government bonds as collateral to finance working capital.
- Tradeoff between tax distortion and output loss.
- Key aggregates are above trend until default.
- Declines in output and investment are in line with data.
- Consistent with both Argentina and Italy.

Appendix

Derivation of Collateral Constraint



m' : stochastic discount factor

W' : next-period value of bank

Derivation of Collateral Constraint

- If bank offers $x \geq \xi(k' + qb')$, lender will accept.
- So if $\ell > \xi(k' + qb')$, bank will choose not to repay and instead offer $\xi(k' + qb')$ to lender.
- Anticipating this behavior, lender will only lend

$$\ell \leq \xi(k' + qb')$$

Go back

Market Clearing

- Market clearing conditions are

$$
\begin{aligned}
e(S) &= 1 \\
n^h(S) &= n^f(S) \\
\ell^h(S) &= \ell^f(S) \\
b'(S) &= B'(S) \\
c(S) + k'(S) &= (1-\delta)k + (1-g)zk^{\alpha} n(S)^{1-\alpha}
\end{aligned}
$$

Go back

Algorithm

The algorithm consists of value function iteration and policy function iteration.

- Create grids for productivity shocks, capital stock and bond holdings.
- Make initial guesses for V^0, $E^{k,0}$, and $E^{b,0}$.
- At each grid point (z, k, b) and for each choice of b', first assume the collateral constraint is binding and solve a system of eight equations (the eight constraints in the value function) with eight unknowns $\{c, n, d, k', \tau, w, q, \mu\}$ using a nonlinear equation solver.
- If the multiplier μ is negative, set it to zero, drop the collateral constraint and solve the system of seven equations with seven unknowns.

Algorithm

- The solutions are the economy's competitive equilibrium conditions if the government does not default and chooses b'.
- In a similar fashion, solve for the economy's competitive equilibrium conditions if the government defaults.
- Given these solutions, calculate the welfare $V^d(z, k, b)$ and $V^r(z, k, b) = \max \widehat{V}^r(z, k, b; b')$, and choose the optimal $b'^* \in \text{ argmax } \widehat{V}^r(z, k, b; b')$.
- Use the results to choose optimal default decision: $D^* = 1$ if $V^d > V^r$ and $D^* = 0$ otherwise.

Algorithm

- Iterate until the value function V converges.
- Update competitive equilibrium conditions and agents' conditional expectations.
- Iterate until the expectations E^k and E^b converge.

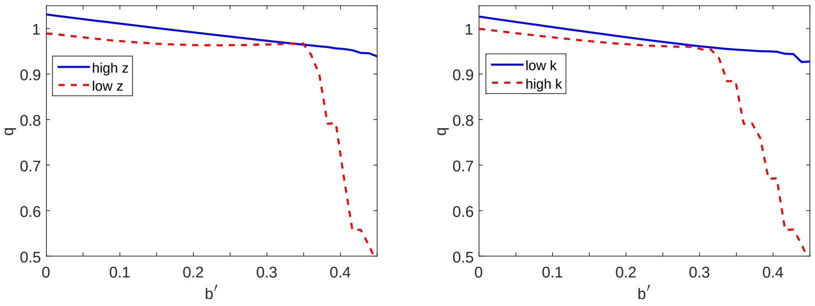
Go back

Argentina: Business Cycle Statistics

	Data	Model
σ_y	5.66%	5.40%
σ_c/σ_y	1.14	0.41
σ_i/σ_y	2.95	3.59
σ_n/σ_y	0.31	0.53
σ_{r^s}	2.51%	1.21%
$corr(y, c)$	0.89	0.51
$corr(y, i)$	0.87	0.96
$corr(y, n)$	0.36	0.60
$corr(y, r^s)$	-0.62	-0.64

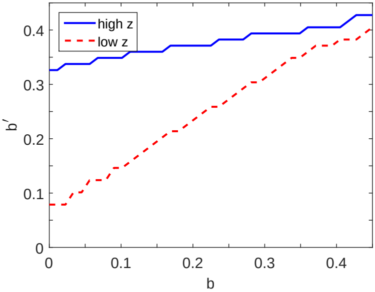
Go back

Bond Price Functions

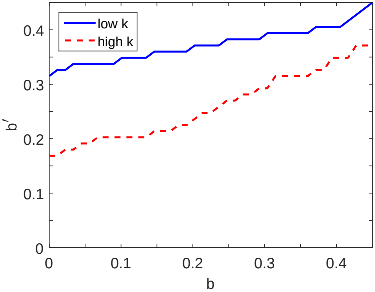


(a) across z (b) across k

Policy Functions for Debt b'

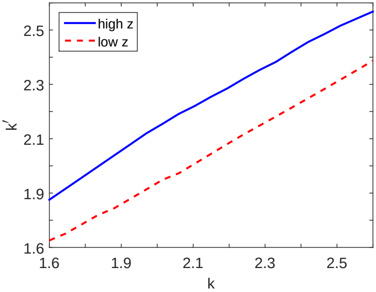


(a) across z

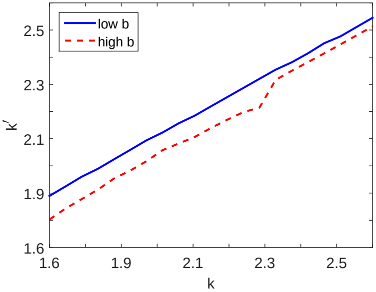


(b) across k

Evolution of Capital k'

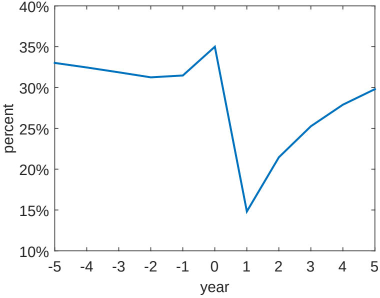


(a) across z

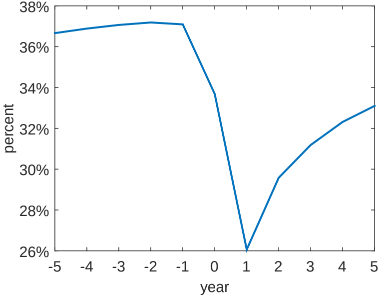


(b) across b

Argentina: Debt and Tax Dynamics around Default



(a) Debt/GDP



(b) Tax Rate