Optimal Monetary Policy and Term Structure in A Continuous-Time DSGE Model

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Outline

- Motivation
- Main Results
- Model
- Empirical Study
- Conclusion and Future Research

Motivation

- **Monetary policy and term structure are intimately related**
  - policy setting short-term (nominal) rate
  - term structure reflects current short-rate and possible future policy changes
- **Literature of Monetary Policy (Macroeconomics)**
  - New Keynesian model: inefficiency caused by sticky price
    - policy can reduce inefficiency, money is not neutral
    - but, no simple, practical optimal policy
    - implications on term structure and asset prices are ignored
  - Taylor rule (Taylor, 1993): a linear combination of macro variables, widely used in policy analysis
    - simple and practical, but not clear whether it is optimal in any equilibrium model
      "Of course, it will surprise no one that such a simple rule is unlikely to correspond to fully optimal policy in the context of a particular economic model."

— Michael Woodford (2001, AER)

Motivation

- **Term Structure Models (Finance)**
  - short rate is a linear combination of latent variables
  - using no-arbitrage to derive term structure
- **Monetary Policy and Term Structure**
  - two huge strands of literature are separated until recently
    - Ang and Piazzesi (2003) use Taylor rule as short rate in a no-arbitrage term structure model
    - Li, Li and Yu (2013), Taylor rule with policy regimes
  - connecting macro variables with term structure through an exogeneous Taylor rule
    - state price is arbitrarily specified, not determined in an equilibrium context
- **Can we derive all the important ingredients in a general equilibrium model?**
  - in which all the followings are determined endogenously
    - optimal monetary policy
    - macro dynamics
    - term structure of interest rate

Motivation

- **We do this in a continuous-time setting**
  - traditionally, New Keynesian model is done in discrete-time
  - continuous-time models proved to be a very useful tools in the literature of asset pricing
    - most term structure and portfolio choice models are in continuous-time
- **Related Literature**
  - optimal monetary policy
    - New Keynesian model in discrete time
    - "optimal policy" is **Not** simple and time-inconsistent, Galí (2008)
    - discretion or commitment
  - no-arbitrage term structure with Taylor rule
    - Ang and Piazzesi (2003)
    - Li, Li and Yu (2013) with regime-switching Taylor rule
  - New Keynesian model with a generalized Taylor rule
    - Bekaert, Cho and Moreno (2010)
  - deterministic continuous-time model
    - Werning (2012), Cochrane (2014)

Main Results

- **Developed a continuous-time New Keynesian model**
  - macroeconomic variables – inflation and output gap have affine dynamic structure
- **Optimal monetary policy**
  - optimal monetary policy is linear
    - coefficient on output gap is **negative** – a distinction with Taylor rule
  - term structure has closed-form solution
- **Empirical Study**
  - two monetary policy regimes: one is relatively stabilizing than the other
  - monetary policy plays important role in stabilizing macro dynamics

Model

- **A linear production economy in continuous-time**
- **Households: consume final good and supply labor, own firms**
  - price-taker in both good and wage
- **Final good producer: competitive**
  - price-taker in both inputs and output
- **Intermediate good producers: monopolistically competitive**
  - set nominal price for goods produced , but
  - do not adjust good price continuously
- **Monetary authority: set monetary policy (short-term nominal interest rate)**

Model: Households

- **Households choose consumption and labor to maximize utility**

$$u(C,N) = E\left[\int_0^\infty e^{-\rho t}\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \phi\frac{N_t^{1+\eta}}{1+\eta}\right)dt\right],$$

s.t. a budget constraint

$$E\left[\int_0^\infty \xi_t(P_tC_t - W_tN_t)\,dt\right] \leq M_0$$

  - ξ_t is state price density

$$\xi_t = \exp\left[-\int_0^t\left(r_s + \frac{1}{2}\|\lambda_s\|^2\right)ds - \int_0^t \lambda_s \cdot dZ_s\right],$$

Model: Households

- Z stack of shocks to productivity, output gap, aggregate price level, and rate of inflation
- Martingale Approach, Cox and Huang (1989)
  - a common method widely used in asset pricing
  - no need for Markovian structure, e.g., no restrictions on interest rate
- FOCs for households:

$$e^{-\rho t} C_t^{-\gamma} = \psi \xi_t P_t, \qquad e^{-\rho t} \phi N_t^{\eta} = \psi \xi_t W_t$$

Model: Final Good Producer

- price-taker and face perfect competition
- final good is produced from a continuum of intermediate goods, indexed by $i \in [0,1]$, as

$$Y_t = \left(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

- firm chooses C_{it} to maximize profit

$$\max_{\{C_{it}\}} \left\{ P_t Y_t - \int_0^1 P_{it} C_{it} di \right\}$$

- demand for good i

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} Y_t,$$

- final good price index (zero profit condition)

$$P_t = \left(\int_0^1 P_{it}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$$

Model: Firms and Monopolistic Competition

- **Intermediate Good Producers**
  - monopolistically competitive, the only producer of intermediate good i
  - good i is necessary but influence on final good is small
- Firm i produces $Y_{it} = A_t N_{it}$
  - A_t – productivity, and $a_t = \ln A_t$ follows

$$da_t = \mu_a dt + \sigma_a \cdot dZ_t$$

- Firm's decision: two-stage
  - given demand C_{it} and wage W_t, firm chooses labor to minimize costs

$$\min_{N_{it}} \left\{ \frac{W_t}{P_t} N_{it} + \varphi_t (Y_{it} - A_t N_{it}) \right\}$$

  - this yields the real marginal costs of production

$$\varphi_t = \frac{W_t}{P_t} \frac{1}{A_t}$$

Model: Firms and Monopolistic Competition

- In equilibrium, $Y_t = C_t$, $Y_{it} = C_{it}$, and

$$\int_0^1 N_{it} di = N_t$$

  - using FOCs of Households

$$\varphi_t = \phi \frac{Y_t^{\gamma+\eta}}{A_t^{1+\eta}} H_t^{\eta}$$

where

$$H_t = \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} di > 1$$

Equilibrium under Flexible Price

- **Intermediate goods producers choose prices**

$$\max_{P_{it}} \left(\frac{P_{it}}{P_t} C_{it} - \varphi_t C_{it} \right) = \max_{P_{it}} \left[\left(\frac{P_{it}}{P_t} \right)^{1-\epsilon} - \varphi_t \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} \right] Y_t$$

  - C_{it} is the demand function of final good producer
  - **optimal price**

$$P_{it} = \frac{\epsilon}{\epsilon - 1} \varphi_t P_t$$

  - **using price index**

$$\varphi_t = \bar{\varphi} \equiv \frac{\epsilon - 1}{\epsilon}$$

  - this implies

$$P_{it} = P_t, \quad C_{it} = C_t = Y_t, \quad H_t = 1$$

Equilibrium under Flexible Price

- output (potential output)

$$\ln \bar{Y}_t = \nu \ln A_t - \frac{1}{\gamma + \eta} \ln \frac{\phi}{\bar{\varphi}}$$

  where

$$\nu = \frac{1 + \eta}{\gamma + \eta}$$

- price of risk

$$\bar{\lambda}_t = \gamma \nu \sigma_a$$

- interest rate

$$\bar{r}_t = \rho + \bar{\pi} + \gamma \nu \mu_a - \frac{1}{2} \gamma^2 \|\nu \sigma_a\|^2$$

- money (monetary policy) is neutral

Equilibrium under Sticky Price

- **Sticky Price**: intermediate goods producers adjust output price in a Calvo (1983) fashion
  - each firm adjusts price with probability $1 - e^{-\delta \Delta t}$ from t to $t + \Delta t$
  - thus, firms set price according to

$$\max_{P_{it}} \frac{1}{\xi_t P_t} E_t \left\{ \int_t^\infty e^{-\delta(s-t)} \xi_s P_s \left[\left(\frac{P_{it} e^{\bar{\pi}(s-t)}}{P_s} \right)^{1-\epsilon} - \varphi_s \left(\frac{P_{it} e^{\bar{\pi}(s-t)}}{P_s} \right)^{-\epsilon} \right] Y_s ds \right\}$$

where $e^{-\delta(s-t)}$ is the probability that firm does not adjust price

- **Optimal price set by**:

$$\frac{P_t^*}{P_t} = \frac{E_t \left[\int_t^\infty e^{-(\delta+\rho)(s-t)} Y_s^{1-\gamma} \frac{\varphi_s}{\bar{\varphi}} \left(\frac{P_s}{P_t e^{\bar{\pi}(s-t)}} \right)^{\epsilon} ds \right]}{E_t \left[\int_t^\infty e^{-(\delta+\rho)(s-t)} Y_s^{1-\gamma} \left(\frac{P_s}{P_t e^{\bar{\pi}(s-t)}} \right)^{\epsilon-1} ds \right]}$$

Log-linear Approximation

- price index and Calvo pricing imply

$$d \ln P_t = \left(\delta \frac{P_t^* - P_t}{P_t} + \bar{\pi} \right) dt + \boldsymbol{\sigma_p} \cdot dZ_t$$

  - adding a shock term $\boldsymbol{\sigma_p}$
- this shows inflation (local) is

$$\pi_t \equiv \delta \frac{P_t^* - P_t}{P_t} + \bar{\pi}$$

- linearize the price setting rule at flexible price equilibrium

$$\frac{P_t^* - P_t}{P_t} = (\delta + \tilde{\rho}) E_t \left[\int_t^{\infty} e^{-(\delta+\tilde{\rho})(s-t)} \left(\ln \frac{\varphi_s}{\bar{\varphi}} + \ln \frac{P_s}{P_t} - \bar{\pi}(s-t) \right) ds \right]$$

  - work out $E_t[d\pi_t]$

New Keynesian Philips Curve

- using the expression of φ_t

$$\ln \frac{\varphi_t}{\bar{\varphi}} = (\gamma + \eta) \ln \frac{Y_t}{\bar{Y}_t} + \eta \boldsymbol{\ln H_t}$$

  - taking $\ln H_t \approx 0$

- **New Keynesian Phillips Curve**

$$d\pi_t = \left[\tilde{\rho}\left(\pi_t - \bar{\pi}\right) - \kappa_y \left(\boldsymbol{y_t} - \boldsymbol{\bar{y}_t}\right)\right] dt + \sigma_\pi \cdot dZ_t$$

  - lower case is logarithmic
  - $\tilde{\rho} > 0$, a constant
  - $\kappa_y = \delta(\delta + \tilde{\rho})(\gamma + \eta) > 0$
  - $\boldsymbol{y_t - \bar{y}_t = x_t}$ **– output gap**

Output Gap

- Aggregate demand: rewrite households' FOC on consumption in term Y_t as
  - sticky price

$$dy_t = \frac{1}{\gamma}\left(\boldsymbol{r_t} - \rho - \pi_t + \frac{1}{2}\|\lambda_t\|^2\right) dt + \frac{1}{\gamma}(\lambda_t - \sigma_p) \cdot dZ_t$$

  - flexible price

$$d\bar{y}_t = \nu da_t = \nu \left[\mu_a dt + \sigma_a \cdot dZ_t\right]$$

  - **output gap** $x_t \equiv y_t - \bar{y}_t$ under sticky price

$$dx_t = \frac{1}{\gamma}\left[\boldsymbol{r_t} - \tilde{r} - (\pi_t - \bar{\pi})\right] dt + \sigma_x \cdot dZ_t$$

    - $\boldsymbol{r_t}$ – nominal short-term interest rate
    - $\sigma_x = \frac{1}{\gamma}(\lambda_t - \sigma_p - \gamma v \sigma_a)$
    - $\tilde{r}$ – constant

Optimal Monetary Policy and Term Structure

- **Equilibrium under sticky price is inefficient**

$$d\begin{pmatrix} \boldsymbol{\pi_t} \\ \boldsymbol{x_t} \end{pmatrix} = \begin{pmatrix} \tilde{\rho}\left(\boldsymbol{\pi_t} - \bar{\pi}\right) - \kappa_y \boldsymbol{x_t} \\ \frac{1}{\gamma}\left[\boldsymbol{r_t} - \tilde{r} - \left(\boldsymbol{\pi_t} - \bar{\pi}\right)\right] \end{pmatrix} dt + \begin{bmatrix} \sigma_\pi^\top \\ \sigma_x^\top \end{bmatrix} dZ_t$$

  - non-zero output gap and inflation vary over time
- **Thus, monetary policy can reduce the inefficiency (not neutral)**
  - monetary objective (welfare lost plus control costs)

$$E\left[\frac{1}{2}\int_0^\infty e^{-\rho_m t}\left(\left(\boldsymbol{\pi_t} - \bar{\pi}\right)^2 + \alpha_x \boldsymbol{x_t}^2 + \boldsymbol{\alpha_r}\, \boldsymbol{r_t}^2\right) dt\right]$$

  - choose $\boldsymbol{r_t}$ to minimize objective by steering the equilibrium
  - $\boldsymbol{r_t}$ is the monetary policy instrument
  - ρ_m – monetary discount rate

Optimal Monetary Policy: Single Regime

- **Optimization is standard control problem**
  - solved by dynamic programming (HJB)
  - the cost function

$$V(\pi, x) = \frac{1}{2}\left(A_\pi(\pi - \bar{\pi})^2 + A_x x^2\right) + B_{\pi x} x(\pi - \bar{\pi}) + C_x x + C_\pi(\pi - \bar{\pi}) + D$$

  - the coefficients are determined by 6 algebraic equations
  - $A_x,\ A_\pi > 0,\ B_{\pi x} < 0$
- **Optimal interest rate rule**

$$\begin{aligned} \boldsymbol{r} = -\frac{1}{\alpha_r \gamma}\frac{\partial V}{\partial x} &= -\frac{1}{\gamma \alpha_r}\left(A_x \mathbf{x} + B_{\pi x}(\boldsymbol{\pi} - \bar{\pi}) + C_x\right) \\ &= \beta_x^* x + \beta_\pi^*(\pi - \bar{\pi}) + \beta_0^* \end{aligned}$$

  - the optimal policy is simple and linear
  - coefficient on output gap $\boldsymbol{\beta_x}$ is **negative**

Monetary Policy and Macro Stability

- Given an arbitrary linear policy rule

$$r_t = \beta_0 + \beta_\pi \left(\pi_t - \bar{\pi}\right) + \beta_x \, x_t$$

- dynamics of output gap and inflation:

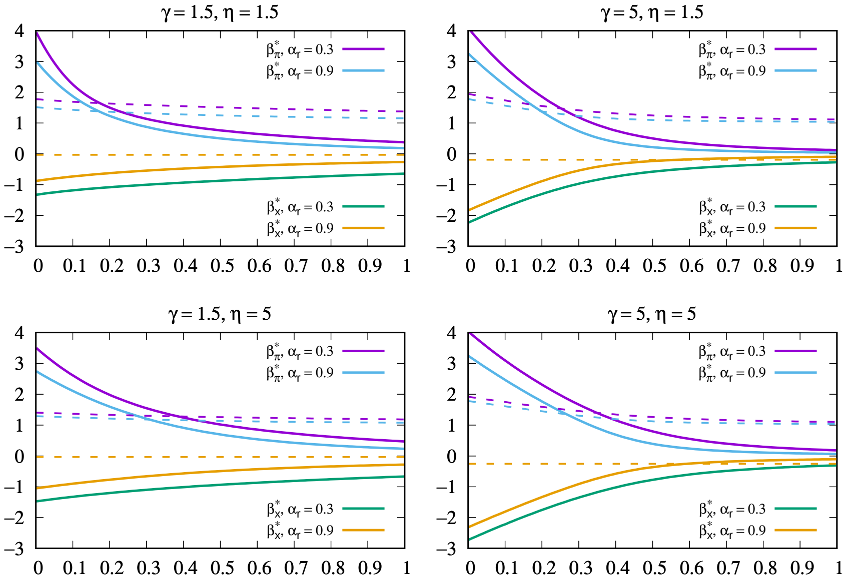
$$d\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \left\{ ? + \underbrace{\begin{bmatrix} \tilde{\rho} & -\kappa_y \\ \frac{1}{\gamma}(\beta_\pi - 1) & \frac{1}{\gamma}\beta_x \end{bmatrix}}_{\boldsymbol{\kappa}} \begin{pmatrix} \pi_t - \bar{\pi} \\ x_t \end{pmatrix} \right\} dt + \begin{bmatrix} \sigma_\pi^\top \\ \sigma_x^\top \end{bmatrix} dZ_t^*$$

- conditions for stability: matrix $\boldsymbol{\kappa}$ has two negative eigenvalues

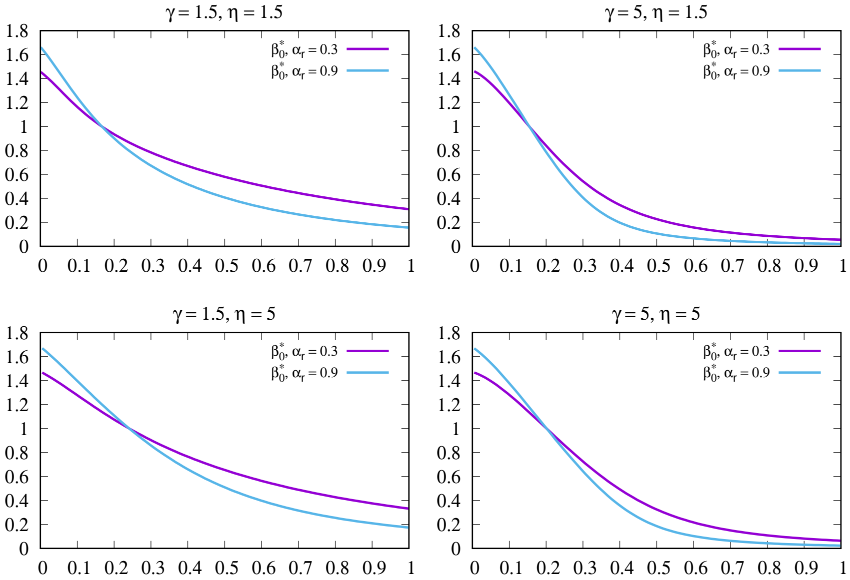
$$\beta_\pi > 1 - \frac{\tilde{\rho}}{\kappa_y}\beta_x, \quad \beta_x < -\gamma\tilde{\rho}$$

- **Taylor principle** is necessary but not sufficient for stability

Coefficients on Inflation and Output Gap vs ρ_m



Constant Term $(\beta_0/\tilde{r})$ vs ρ_m



Optimal Monetary Policy: $\alpha_r = 0$

- Minimizing problem degenerates
- **As** $\alpha_r \to 0$
  - Monetary cost:

$$V(\pi, x) \to \frac{1}{2}\bar{A}_\pi(\pi - \bar{\pi})^2 + \frac{\bar{A}_\pi}{2\rho_m}\|\sigma_\pi\|^2$$

  - Interest rate rule:

$$r^* \to \tilde{r} + \text{term of } \alpha_r^{-1/2} \times \left(x - \frac{\kappa_y \bar{A}_\pi}{\alpha_x}(\pi - \bar{\pi})\right)$$

- **Optimal monetary policy**

$$r^* = \begin{cases} \tilde{r}, & \text{if} \quad x = \frac{\kappa_y \bar{A}_\pi}{\alpha_x}(\pi - \bar{\pi}) \\ \infty, & \text{if} \quad x > \frac{\kappa_y \bar{A}_\pi}{\alpha_x}(\pi - \bar{\pi}) \\ -\infty, & \text{if} \quad x < \frac{\kappa_y \bar{A}_\pi}{\alpha_x}(\pi - \bar{\pi}), \end{cases}$$

Optimal Monetary Policy: K Regimes

- **Suppose monetary discount rate has K possible values**, $\rho_{m1}, \cdots, \rho_{mK}$
  - worry about short-term results – high discount rate
  - care about long-term results – lower discount rate
  - switching among discount rates follows a continuous-time Markov Chain with a constant transition matrix Q
- **Let Λ_{kk} be the k-th eigenvalue** of

$$Q - \begin{bmatrix} \rho_{m1} & \cdots & 0 \\ \cdots & \rho_{mk} & \cdots \\ 0 & \cdots & \rho_{mK} \end{bmatrix}$$

- The **optimal monetary policy under regime k** is the same as the **single-regime one if using $-\Lambda_{kk}$ as the monetary discount rate**

Term Structure of Interest Rate: Single Regime

- **Given a linear interest rate rule**

$$r_t = \beta_0 + \beta_\pi(\pi_t - \bar{\pi}) + \beta_x x_t$$

- **Dynamics of inflation and output gap under risk-neutral measure**

$$d\begin{pmatrix} \pi_t - \bar{\pi} \\ x_t \end{pmatrix} = \left\{\theta + \begin{bmatrix} \tilde{\rho} & -\kappa_y \\ \frac{1}{\gamma}(\beta_\pi - 1) & \frac{1}{\gamma}\beta_x \end{bmatrix}\begin{pmatrix} \pi_t - \bar{\pi} \\ x_t \end{pmatrix}\right\} dt + \begin{bmatrix} \sigma_\pi^\top \\ \sigma_x^\top \end{bmatrix} dZ_t^*$$

where

$$\theta = \begin{pmatrix} -\sigma_\pi \cdot (\sigma_p + \gamma\sigma_x) \\ \frac{1}{\gamma}[\beta_0 - \tilde{r} - \gamma\sigma_x \cdot (\sigma_p + \gamma\sigma_x)] \end{pmatrix}$$

- **Bond prices**

$$\boldsymbol{B(\tau, \pi_t, x_t) = e^{-F(\tau) - G_\pi(\tau)(\pi_t - \bar{\pi}) - G_x(\tau)x_t}}$$

Term Structure with K Monetary Regimes

- **Policy rule follows switching regimes**

$$r_t = \beta_{k0} + \beta_{k\pi}(\pi_t - \bar{\pi}) + \beta_{kx} x_t$$

  - k – index of policy regime
  - hazard rate of policy change — $Q_{kk} < 0$

- **Under policy regime k, macro dynamics is**

$$d \begin{pmatrix} \pi_t - \bar{\pi} \\ x_t \end{pmatrix} = \left\{ \theta_k + \boldsymbol{\kappa_k} \begin{pmatrix} \pi_t - \bar{\pi} \\ x_t \end{pmatrix} \right\} dt + \sigma\, dZ_t^*$$

  - $\boldsymbol{\kappa_k}$ is a matrix

$$\boldsymbol{\kappa_k} = \begin{bmatrix} \tilde{\rho} & -\kappa_y \\ \frac{1}{\gamma}(\beta_{k\pi} - 1) & \frac{1}{\gamma}\beta_{kx} \end{bmatrix}$$

Term Structure with K Monetary Regimes

- **Bond price under regime k**

$$e^{-F_k(\tau)-(\pi_t-\bar{\pi},\, x_t)G_k(\tau)},$$

  - F_k and G_k satisfy a system ODEs (after first-order approximation as in Li, Li and Yu (2013))

$$F_k' - \theta_k^\top G_k + \frac{1}{2}\textbf{Trace}\left(\sigma^\top G_k G_k^\top \sigma\right) - \beta_{k0} + \sum_{n=1}^{N} Q_{kn} e^{-F_n+F_k} = 0$$

$$G_k' + \kappa_k^\top G_k - \sum_{n=1}^{N} Q_{kn} e^{-F_n+F_k}[G_n - G_k] - \beta_k = \mathbf{0},$$

$$\theta_k = \begin{pmatrix} -\gamma\sigma_\pi\sigma_{x\pi} \\ \frac{1}{\gamma}\left(\beta_{k0} - \tilde{r} - \gamma^2\|\sigma_x\|^2\right) \end{pmatrix},\ \sigma = \begin{bmatrix} \sigma_{\pi\pi} & 0 \\ \sigma_{x\pi} & \sigma_{xx} \end{bmatrix},\ \beta_k = \begin{pmatrix} \beta_{k\pi} \\ \beta_{kx} \end{pmatrix}$$

  - **these ODEs collapsed to the exact ODEs with single-regime when $q_{kk} = 0$**

Empirical Study

- **Use the model to analyze US monetary policy**
  - term structure data
- **Empirical questions**
  - is past US monetary policy optimal relative to our model
  - any monetary policy regime change
    - policy changes by political pressure
    - use different ρ_m
    - also motivated by previous studies, e.g., Taylor (1993) and Li, Li and Yu (2013)
  - any stabilizing or destabilizing policy
- **No restrictions on the coefficients of policy rule in estimation**
  - optimal policy is linear
  - enable to check whether a policy is optimal after the estimation

Empirical Method

- **MCMC method as used in Li, Li and Yu (2013)**
  - transition probability of inflation and output gap in physical measure

$$d\begin{pmatrix}\pi_t \\ x_t\end{pmatrix} = \left\{ \begin{pmatrix} 0 \\ \frac{\beta_{k0}-\tilde{r}}{\gamma} \end{pmatrix} + \boldsymbol{\kappa_k} \begin{pmatrix} \pi_t - \bar{\pi} \\ x_t \end{pmatrix} \right\} dt + \sigma\, dZ_t$$

  - differences between data and model yields are iid normal

$$R_t(\tau_m) = \hat{R}_{k_t,t}(\tau_m) + \epsilon_{m,t}, \quad \epsilon_{m,t} \sim N(0, \sigma_m)$$

    - pricing errors $\epsilon_m \sim N(0,\ \sigma_m)$
    - model yield

$$\hat{R}_{k,t}(\tau_m) = \frac{F_k(\tau_m) + (\pi_t - \bar{\pi},\ x_t)G_k(\tau_m)}{\tau_m}$$

Data

- Term structure data from CRSP
  - Fama-Bliss Treasury yields with maturities: 3 months, 1, 2, 3, 4, and 5 years
- Inflation and GDP
  - GDP deflator and GDP from St. Louis Fed
  - output gap is HP filtered log GDP
- Sample periods 1952.2 - 2007.3 quarterly
  - the same sample period as in Li, Li and Yu (2013)

Empirical Results

- **Single-regime model**
  - policy

$$r_t = \underset{(0.0007)}{0.0537} + \underset{(0.0286)}{\mathbf{0.8036}}\,(\pi_t - 0.03) + \underset{(0.0304)}{\mathbf{0.0343}}\,x_t$$

    - coefficient on output gap is **positive** — cannot be optimal
  - implied macro dynamics

$$d\begin{pmatrix}\pi_t\\ x_t\end{pmatrix} = \left\{\begin{pmatrix}0\\ \underset{(0.0027)}{-0.0007}\end{pmatrix} + \begin{bmatrix}\underset{(0.0027)}{0.0096} & \underset{(0.0105)}{-0.0104}\\ \underset{(0.0681)}{-0.3513} & \underset{(0.0594)}{0.0649}\end{bmatrix}\begin{pmatrix}\pi_t - 0.03\\ x_t\end{pmatrix}\right\} dt + ?dZ_t$$

    - macro dynamics is unstable
    - the eigenvalues of κ-matrix are -0.0292 and **0.1037**

Empirical Results

- **Two Monetary Policy Regimes**
  - policy

$$r_t = \begin{cases} \underset{(0.0010)}{0.0756} + \underset{(0.0767)}{\mathbf{1.1350}}\,(\pi_t - 0.03) - \underset{(0.0743)}{\mathbf{0.1650}}\,x_t, & \text{Regime 1,} \\ \underset{(0.0009)}{0.0398} + \underset{(0.0225)}{\mathbf{0.6511}}\,(\pi_t - 0.03) + \underset{(0.0373)}{\mathbf{0.2772}}\,x_t, & \text{Regime 2.} \end{cases}$$

    - **might be optimal in regime 1 but not in regime 2**
  - macro dynamics

$$d\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \left\{ \begin{pmatrix} 0 \\ \underset{(0.0012)}{-0.0051} \end{pmatrix} + \begin{bmatrix} \underset{(0.0072)}{\mathbf{0.0548}} & \underset{(0.0197)}{-\mathbf{0.0276}} \\ \underset{(0.0219)}{\mathbf{0.0364}} & \underset{(0.0220)}{-\mathbf{0.0447}} \end{bmatrix} \begin{pmatrix} \pi_t - 0.03 \\ x_t \end{pmatrix} \right\} dt + ?dZ_t$$

$$d\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \left\{ \begin{pmatrix} 0 \\ \underset{(0.0009)}{-0.0148} \end{pmatrix} + \begin{bmatrix} \underset{(0.0072)}{\mathbf{0.0548}} & \underset{(0.0197)}{-\mathbf{0.0276}} \\ \underset{(0.0136)}{-\mathbf{0.0937}} & \underset{(0.0151)}{\mathbf{0.0747}} \end{bmatrix} \begin{pmatrix} \pi_t - 0.03 \\ x_t \end{pmatrix} \right\} dt + ?dZ_t$$

    -

Table: **Regression Analysis of Observed Yields on Model Yields**

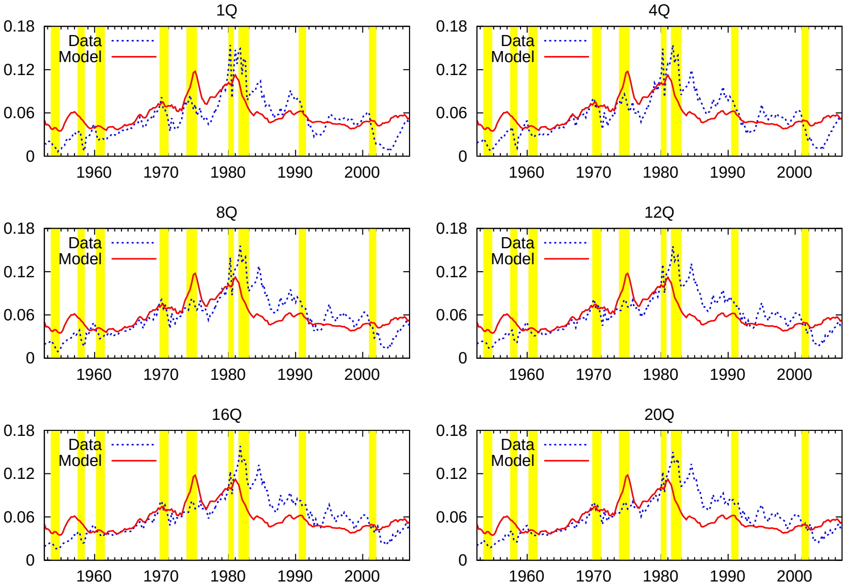
This table provides the regression analysis of observed zero-coupon government bond yields on model-implied yields under the single-regime model at different maturities. The regression equation is

$$\text{Observed Yields} = \gamma_0 + \gamma_1 \text{Model Yields} + \text{error},$$

where the model yields are computed based on estimated parameters in the previous table. Standard errors are reported in parentheses. The maturities of the bonds range from one quarter (1Q) to five years (20Q).

Bond Maturity	Single Regime				Two Regimes			
	γ_0	γ_1	Standard Deviation of Residuals	$\mathbf{R}^2$	γ_0	γ_1	Standard Deviation of Residuals	$\mathbf{R}^2$
1Q	-0.0102 (0.0046)	1.0635 (0.0760)	0.0208	47.3%	-0.0087 (0.0022)	1.0768 (0.0374)	0.0131	79.2%
4Q	-0.0052 (0.0047)	1.0472 (0.0777)	0.0213	45.5%	-0.0056 (0.0021)	1.0839 (0.0348)	0.0123	81.7%
8Q	-0.0009 (0.0048)	1.0061 (0.0786)	0.0215	42.9%	-0.0035 (0.0020)	1.0666 (0.0320)	0.0115	83.6%
12Q	0.0039 (0.0047)	0.9531 (0.0784)	0.0214	40.4%	-0.0002 (0.0020)	1.0213 (0.0314)	0.0115	82.9%
16Q	0.0067 (0.0048)	0.9261 (0.0786)	0.0215	38.9%	0.0018 (0.0020)	0.9909 (0.0316)	0.0117	81.9%
20Q	0.0091 (0.0047)	0.8999 (0.0778)	0.0213	38.1%	0.0037 (0.0021)	0.9534 (0.0315)	0.0119	80.7%
mean	0.0006	0.9827	0.0213	**42.2%**	-0.0021	1.0322	0.0120	**81.7%**

Data and Model-Implied Yields (Single-Regime)



Data and Model-Implied Yields (Two-Regime)

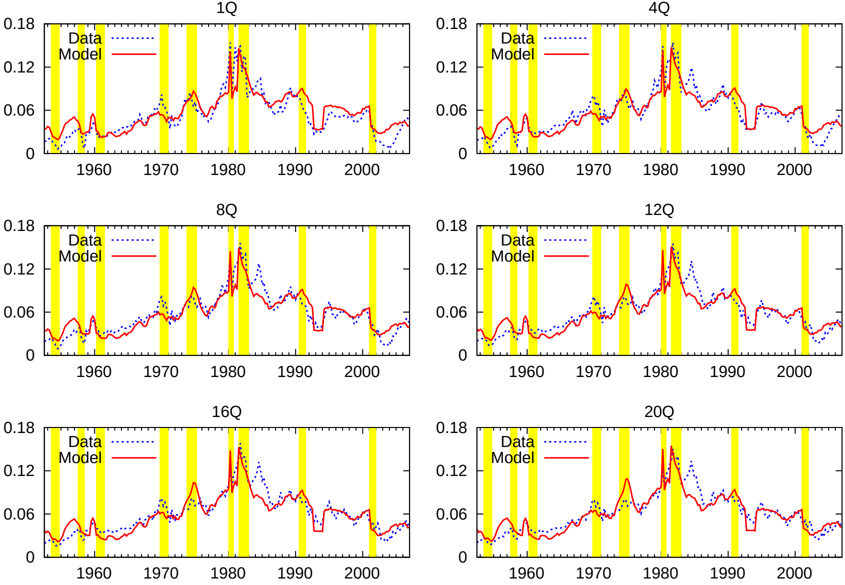


Table: **Parameter Estimates**

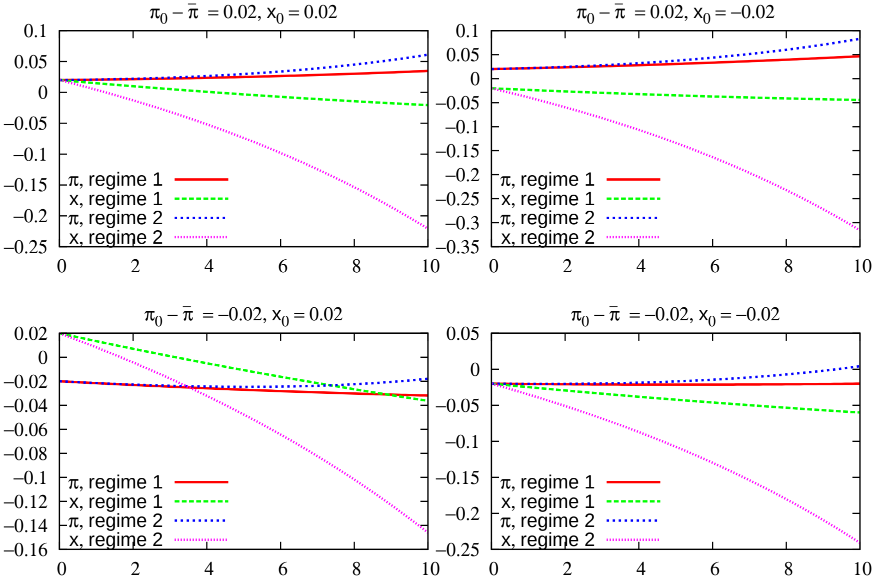
This table reports the empirical estimates of the model parameters for the single-regime model. We run MCMC with 100,000 iterations and use the posterior mean (standard deviation) of the last 50,000 iterations as estimates of the model parameters (standard error, shown in parentheses).

Parameter	Single Regime		Two Regimes			
	-		Regime 1		Regime 2	
ρ	0.0159	(0.0007)	0.0168	(0.0016)		
γ	**0.5808**	(0.1502)	**3.7910**	(0.5369)		
$\nu\mu_a$	0.0148	(0.0017)	0.0136	(0.0006)		
κ_y	0.0104	(0.0105)	0.0276	(0.0197)		
$\sigma_{\pi\pi}$	0.0080	(0.0004)	0.0083	(0.0004)		
σ_{xx}	0.0176	(0.0008)	0.0192	(0.0009)		
$\sigma_{x\pi}$	-0.0015	(0.0007)	-0.0047	(0.0010)		
β_0	0.0537	(0.0006)	0.0756	(0.0010)	0.0398	(0.0009)
β_π	**0.8036**	(0.0286)	**1.1350**	(0.0767)	**0.6511**	(0.0225)
β_x	**0.0343**	(0.0304)	**-0.1650**	(0.0743)	**0.2772**	(0.0373)
Q_{kk}	-	-	-0.0184	(0.0080)	-0.1433	(0.0248)
$\sigma_{m=1,\cdots,6}$	0.0217	(0.0011)	0.0134	(0.0007)		
	0.0213	(0.0010)	0.0122	(0.0006)		
	0.0214	(0.0010)	0.0115	(0.0006)		
	0.0213	(0.0010)	0.0116	(0.0006)		
	0.0215	(0.0010)	0.0120	(0.0006)		
	0.0215	(0.0010)	0.0122	(0.0006)		

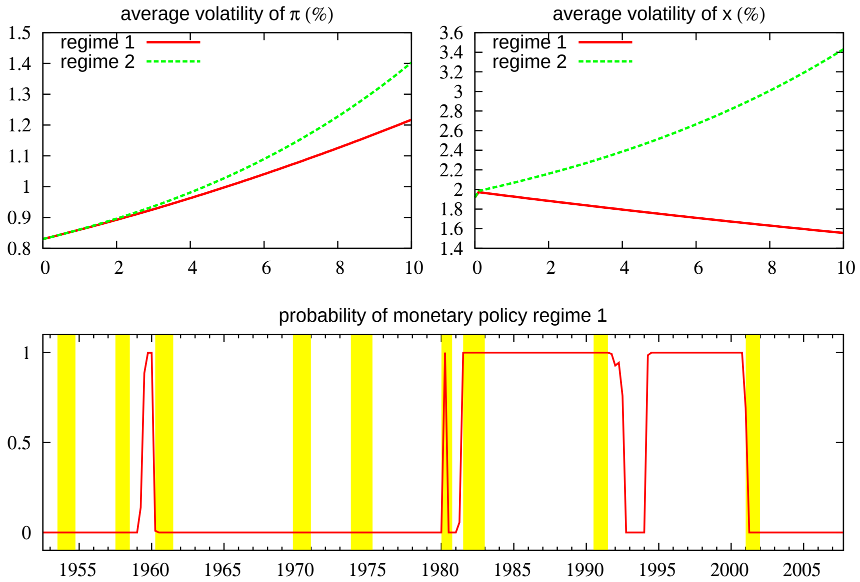
Summary Empirical Results

- **Single-regime vs Two-regime**
  - parameter estimates
    - two-regime is better
  - explaining observed term structure
    - R^2 is much better with two-regime model
    - two-regime is better in time-series
  - optimality
    - regime 1 in two-regime model can be optimal but with very large monetary discount rate
    - not optimal in single-regime model
- **These results are roughly aligned with Li, Li and Yu (2013) in a No-arbitrage Framework**
- **Macro Stability in Two-regime Model**

Expected Future Gaps



Average Volatility of Gaps and Estimated Regimes



Optimality of Estimated Policy Under Regime 1

Table: **Fitted Parameters of Monetary Loss Function and Implied Optimal Policy Coefficients**

ρ_m	α_x	α_r	β_0	β_π	β_x
0.197	0.000	0.979	0.043	1.120	-0.238
			[0.074, 0.078]	[0.985, 1.285]	[-0.311, -0.019]

The optimal policy coefficients βs are based on single-regime model. The parentheses are the 95% confidence intervals of the estimates under regime 1 in the two-regime model.

- **monetary loss function**

$$\frac{1}{2}\int_0^\infty e^{-\boldsymbol{\rho_m t}}\left[(\pi_t-\bar{\pi})^2+\boldsymbol{\alpha_x}\,x_t^2+\alpha_r\,r_t^2\right]dt$$

- **large monetary discount rate**
- **small coefficient on output gap in loss function**
- **large constant coefficient of estimated policy**

Conclusion and Future Research

- **Developed a continuous-time New Keynesian model**
  - optimal monetary policy is linear
  - affine macro dynamics
  - term structure has closed-form solution without policy regimes
  - or simple approximation with policy regimes
- **Empirical studies**
  - two policy regime: one is more stabilizing than the other
  - none of them achieve absolute stability
  - more stable one is near-optimal
- **The model can be extended in several directions**:
  - incorporating preference shock or state-dependent preferences
  - international aspects, exchange rate?