Employment, Wages and Optimal Monetary Policy

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Policymaking with multiple reference models

We study an environment in which multiple models provide a good approximation to the true data-generating process.

How can policymakers resolve the tensions between the normative recommendations of these models?

Two models of the labor market

We consider two New Keynesian models that differ only with regard to the details of the labor market:

- search and matching frictions (*s&m*) as in Diamond (1982), Mortensen (1982), and Pissarides (1985),
- sticky nominal wages (*sw*) as in Erceg, Henderson, and Levin (2000).

We argue that policies should:

- be biased towards the recommendations from the sticky wage model
- even if the policymaker views the search and matching model more likely to be the true data-generating process.

Positive versus normative implications

For reasonable parameter choices the two models match existing empirical evidence under a simple instrument rule (near observational equivalence).

But, when policy is set optimally within each model:

- price inflation is stabilized and wage inflation moves to adjust real wages in the search and matching model,
- wage inflation is stabilized and price inflation moves to adjust real wages in the sticky wage model.

The problem and our approach

Objective of the central bank:

$$\min E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L\left(y_{t-1}, y_t, y_{t+1}, prob_t\right) \right\}$$

$prob_t$ is the probability distribution over models held by the policymaker.

Observations of y_t could change $prob_t$ over time (posterior beliefs).

But, we consider two simplifications:

- Experiment I: policymaker assumes model 1 to be true, when in fact model 2 it true.
- Experiment II: policymaker has a time-invariant probability distribution over models and fixes a policy rule at the beginning of time.

Experiment I

If the policymaker implements in the search and matching model the policy that is optimal in the sticky wage model:

- wage inflation is stabilized,
- price inflation moves to adjust real wages,
- welfare is *moderately* lower than under the *s&m*-specific optimal policy.

If the policymaker implements in the sticky wage model the policy that is optimal in the search and matching model:

- price inflation is stabilized,
- wage inflation moves to adjust real wages,
- welfare is *significantly* lower than under the *sw*-specific optimal policy.

We export the optimal monetary policy from one model to the other using optimal targeting rules.

Experiment II

The policymaker holds a time-invariant belief distribution over models.

The coefficients of a simple instrument rule are chosen optimally given the policymaker's preferences over the risk of model misspecification.

The optimal simple rule given the belief distribution:

- closely resembles the optimal commitment policy derived in the sticky wage model
- as long as the sticky wage model is viewed moderately likely (prob $\geq 10\%$)
- reflecting the relatively higher welfare costs of extreme misspecification in experiment I.

Related papers

Our work builds on Levin and Williams (2003), Levin, Wieland, and Williams (2003), Brock, Durlauf and West (2005), Taylor and Wieland (2012).

Cogley and Sargent (2005) and Svensson and Willilams (2005) are closely related.

Yet it differs from these earlier contributions:

- Model-consistent loss functions, not ad hoc loss functions. This really matters!
- The two models fit the same empirical evidence under empirical interest rate rules.
- We focus on labor market aspects showing the importance of smoothing wage inflation.

Brief description of the models

In both models:

- a representative household maximizes the discounted utility of its members subject to a budget constraint,
- wholesale firms produce non-differentiated inputs using labor only taking all prices as given,
- retail firms differentiate the wholesale output and set prices using Calvo (1983) contracts,
- time is discrete.

The two models differ with regard to the labor market.

Household

Standard search and matching model of the labor market:

- a member of the household is either employed or unemployed,
- hours worked are variable if employed,
- employed members earn the nominal wage $P_t w_t$,
- unemployed members obtain benefits in the amount $P_t b^u$,
- total number of job searchers u_t,
- there is perfect risk sharing within the family.

household problem *s&m* model

Firms and labor matching

Wholesale firms employ labor only.

To hire a worker, the firm must first post a vacancy; total vacancies v_t.

Matching function determines number of newly created jobs:

$$m_t = M\left(u_t, v_t\right).$$

When a match occurs, Nash bargaining between the worker and the firm determines real wages and hours worked. Nash bargaining

An existing match survives into the next period with fixed probability.
linearized $s\&w$ model

Household

Standard sticky wage model:

- each household offers differentiated labor services,
- the household chooses his labor supply and the nominal wage using Calvo (1983) contracts under monopolistic competition,
- households share risk perfectly through financial markets,
- a labor bundler aggregates the differentiated labor services into a final labor input,
- wholesale firms purchase labor input.

household problem *sw* model

linearized *sw* model

Impulse response function matching

Assume that in the data monetary policy follows a simple instrument rule:

$$i_t = \rho_R i_{t-1} + (1-\rho_R)\rho_\pi \pi_t.$$

We parameterize each model through impulse response function matching:

$$\hat{\theta} = \arg\min_\theta \left(G - G(\theta, \theta^f)^{model}\right)' \Omega^{-1} \left(G - G(\theta, \theta^f)^{model}\right).$$

The data G are the IFRs to a neutral technology shock from an SVAR as in Christiano, Eichenbaum and Trabandt (2017).

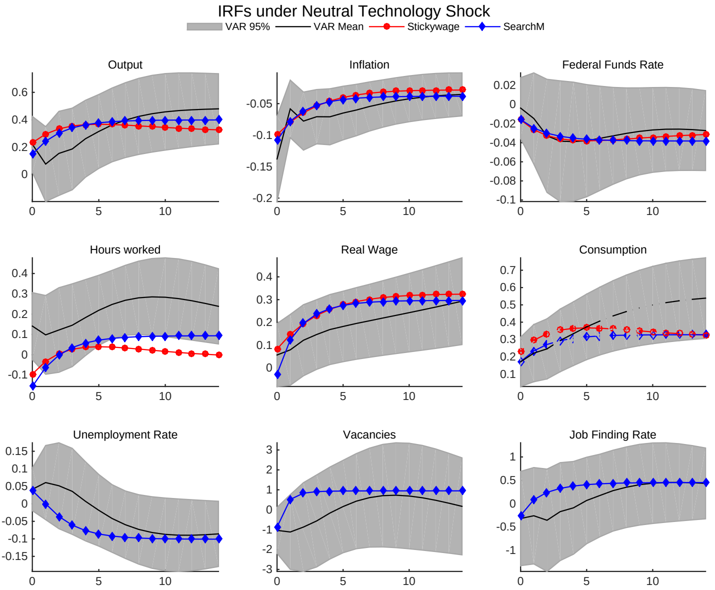

IRFs under Neutral Technology Shock
VAR 95%
VAR Mean
Stickywage
SearchM
Output
Inflation
Federal Funds Rate
Hours worked
Real Wage
Consumption
Unemployment Rate
Vacancies
Job Finding Rate

Table: Parameter estimates

Description	Estimated Parameter	Search Model	Stickywage Model
Interest rate smoothing	ρ_R	0.8555	0.8379
		[0.0294]	[0.0448]
Weights on inflation	ρ_π	1.0003	1.0005
		[0.0001]	[0.0002]
Std technology shock	σ_z	0.0031	0.0033
		[0.0002]	[0.0002]
Habit persistence	h	0	0
		[0.4608]	[0.4140]
Replacement ratio	r^b	0.5345	
		[0.0184]	

Optimal monetary policy with correct model

The policymaker:

- knows the correct model
- maximizes household utility under commitment.

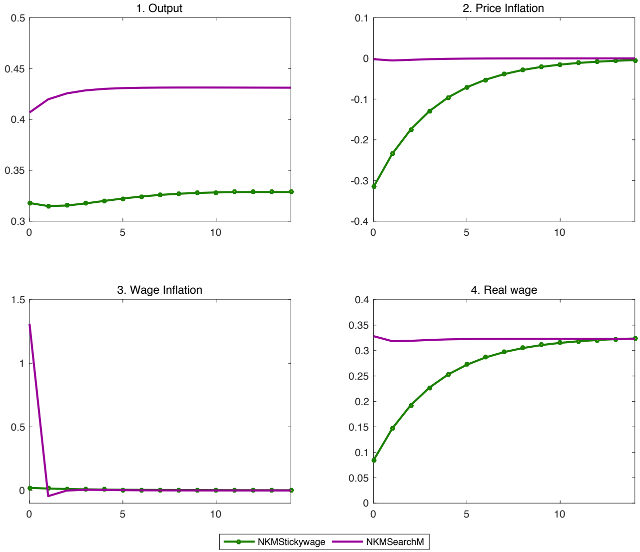
After a positive technology shock in the search and matching model:

- Monetary policy cannot address the inefficiencies from labor matching.
- But policy can curb welfare-costly price dispersion from sticky prices.
- Price inflation is kept near steady state.
- Nominal wages rise to facilitate the desired rise in the real wage.

After a positive technology shock in the sticky wage model:

- Monetary policy trades-off the dispersion of prices *and* wages.
- Wage inflation is kept near steady state.
- Price inflation drops to facilitate the desired rise in the real wage.

Optimal monetary policy with correct model: tech. shock



Extreme model misspecification and robustness

What are welfare implications of implementing in the search and matching model the policy that is optimal in the sticky wage model?

We use optimal targeting rules (Giannoni and Woodford (2003, 2016) and Svensson and Woodford (2004)) to "export" the optimal monetary policy from one model to another model.

The optimal targeting rule specifies the variables in a single target criterion that seeks to implement the optimal monetary policy. We turn:

- the nonlinear optimal policy problem into an LQ problem,
- derive the first order conditions,
- obtain a single condition by substituting out the Lagrange multipliers.

We represent the models in common variables and exchange the rules.

Optimal targeting rule: a simple example

$$\max_{\{\pi_t, x_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{1}{2} \beta^t \left\{ \pi_t^2 + \lambda x_t^2 \right\}$$

$$s.t. \qquad \pi_t = \kappa x_t + \beta E_t \pi_{t+1}$$

with the first order conditions

$$\lambda x_t - \mu_t \kappa = 0$$

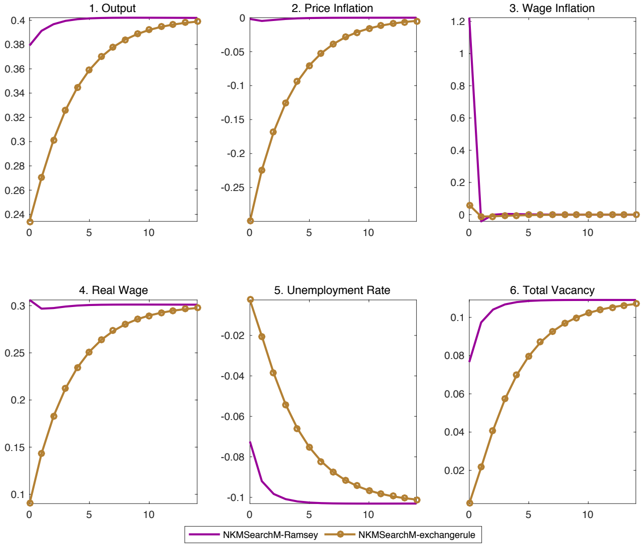
$$\pi_t + \mu_t - \mu_{t-1} = 0$$

and the optimal targeting rule

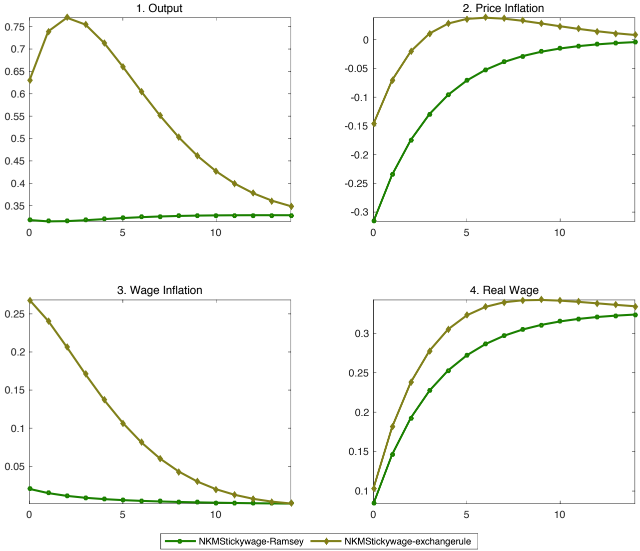
$$\pi_t = -\frac{\lambda}{\kappa} \left(x_t - x_{t-1} \right).$$

optimal targeting rule *s&m* model | optimal targeting rule *sw* model

Exchanging optimal targeting rules: search and matching



Exchanging optimal targeting rules: sticky wages



Exchanging optimal targeting rules: welfare losses

Let the model feature a unit root technology shock and a transitory cost push shock.

Welfare loss of implementing the wrong optimal targeting rules:

- if the true model is the search and matching model: $CEV^{s\&m} = 0.1133$,
- if the true model is the sticky wage model: $CEV^{sw} = 1.3033$.

The optimal targeting rule of the search and matching model is less robust under the preferences of the policymaker.

common simple loss function

Description of the experiment

Our second set of experiments assumes that:

- The private sector knows the correct model.
- Policymakers do not know the correct model.
- Policymakers adopt a simple instrument rule:

$$i_t = \rho_R i_{t-1} + \rho_\pi \pi_t + \rho_\pi^w \pi_t^w + \rho_x x_t$$

  that is common across models.
- Policymakers do not update probability beliefs over models.
- Policymakers never identify the true data-generating process.
- Choice of policy under a veil of uncertainty without revisitation.

Case of model averaging

Preferences under model averaging (or Bayesian) approach:

$$\mathcal{L}^{ma}(\Theta) = \omega * \mathcal{L}^{s\&m}(\Theta) + (1-\omega) * \mathcal{L}^{sw}(\Theta)$$

ω is the time-invariant probability of the $s\&m$ model being true.

The policymaker:

- chooses the coefficients in the simple rule, $\Theta = \left\{ \rho_R, \rho_\pi, \rho_\pi^w, \rho_x \right\}$,
- to minimize the ω-weighted expected loss over the two models,
- when policy adopts the same rule in both models.

Case of minmax

Preferences under minmax approach:

$$\mathcal{L}^{minmax}(\Theta) = \max\left\{\mathcal{L}^{s\&m}(\Theta), \mathcal{L}^{sw}(\Theta)\right\} \tag{1}$$

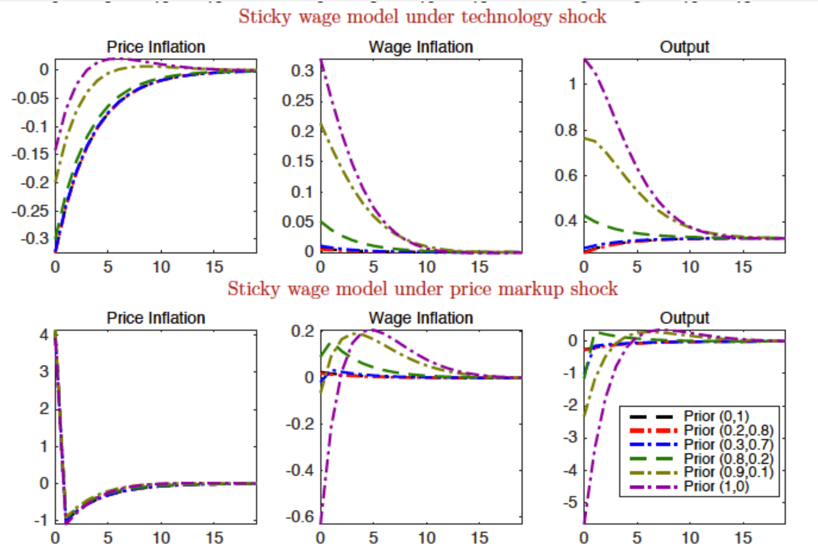
The policymaker:

- chooses the coefficients in the simple rule, $\Theta = \left\{\rho_R, \rho_\pi, \rho_\pi^w, \rho_x\right\}$,
- to minimize the maximum expected loss across the two models,
- when policy adopts the same rule in both models.

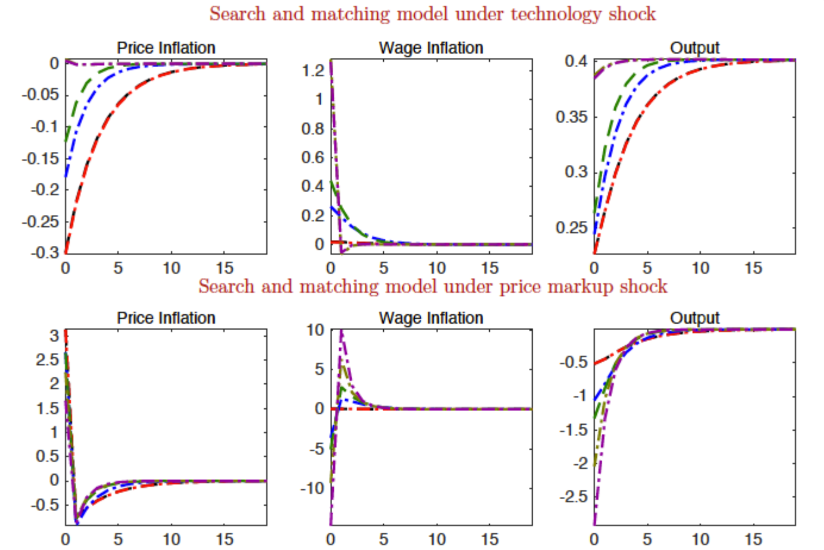
Table: Optimal simple rules: transitory cost push shock $\rho_u = 0$

Approach	Prior	Estimated Taylor-type Rule					Welfare Loss (CEV)	
		ρ_R	ρ_π	ρ_π^w	ρ_x	ratio: $\frac{\rho_\pi}{\rho_\pi^w}$	Search and matching	Sticky Wage
Bayesian	$(0,1)$	0	0	66.6844	2.3852	0	-0.1094	-0.0010
	$(0.1,0.9)$	0	0	61.5860	2.0019	0	-0.1092	-0.0010
	$(0.2,0.8)$	0	0	56.4038	1.6763	0	-0.1091	-0.0011
	$(0.3,0.7)$	0	0.6240	0.5226	0	1.1940	-0.0554	-0.0149
	$(0.4,0.6)$	0	0.6368	0.5160	0	1.2341	-0.0551	-0.0151
	$(0.5,0.5)$	0	0.6558	0.5131	0	1.2781	-0.0548	-0.0153
	$(0.6,0.4)$	0	0.7005	0.5158	0	1.3581	-0.0540	-0.0162
	$(0.7,0.3)$	0	0.8135	0.5231	0	1.5552	-0.0520	-0.0202
	$(0.8,0.2)$	0	1.1725	0.5245	0	2.2355	-0.0446	-0.0438
	$(0.9,0.1)$	0.8177	0.8860	0	0	∞	-0.0149	-0.2061
	$(1,0)$	0.9366	2.1197	0	0	∞	-0.0003	-0.9814
Minimax		0	0	66.6844	2.3852	0	-0.1094	-0.0010
Exchanging OTR							-0.1133	-1.3033

Impulse responses: sticky wages



Impulse responses: search and matching



Sensitivity

We investigate sensitivity to

- restricted optimal simple rules (estimated rule is optimal if $0.6 \leq \omega \leq 0.8$),
- Taylor (1993) or Taylor (1999) rules are not recovered as optimal even under restricted rules,
- habit persistence (slightly weakens finding),
- persistent markup shocks (strengthens findings).

Conclusion

Even if it is difficult/impossible to select the "right" model using statistical decision criteria, it is possible to choose good policies.

By taking into account explicitly the uncertainty over the true data-generating process, the policymaker biases policy towards the recommendations from the sticky wage model (policy instead of model selection).

Only when the policymaker attaches low (or no) probability to the sticky wage model being true—depending on the policymakers attitude towards the risk of model misspecification—does policy reflect the recommendations from the search and matching model.

Household

$$\max_{\tilde{W}_t(j)} E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \left[(1+\bar{\tau}_w) \frac{c_{t+s}^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} h_{t+s}(j)^{1+\phi} \right]$$

$s.t.$

$$P_{t+s} c_{t+s} + B_{t+s+1} = W_{t+s}(j) h_{t+s}(j) + R_{t+s-1} B_{t+s} + T_{t+s}$$

$$h_{t+s}(j) = \left(\frac{\tilde{W}_t(j) \bar{\pi}^s}{W_{t+s}} \right)^{-\frac{\lambda_w}{\lambda_w - 1}} h_{t+s}$$

Back to presentation.

Linear approximation: sticky wage model

Linearizing the equilibrium conditions of the sticky wage model delivers the familiar set of relationships:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^*\right)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p \left(\hat{w}_t - a_t\right) + \hat{\theta}_{p,t}$$

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w \left(\left(\sigma_L + \sigma_C\right) x_t + a_t - \hat{w}_t\right)$$

$$\hat{w}_t = \hat{w}_{t-1} + \pi_t^w - \pi_t$$

Back to presentation.

Optimal targeting rule in search and matching model: LQ

The quadratic loss function can be shown to satisfy:

$$\begin{aligned}\mathcal{L}_t &= P_{\pi,\pi}\pi_t^2 + P_{y,y}\hat{y}_t^2 + P_{n,n}\hat{n}_t^2 + P_{n^-,n^-}\hat{n}_{t-1}^2 + P_{y,n}\hat{n}_t\hat{y}_t + P_{y,n^-}\hat{y}_t\hat{n}_{t-1} \\ &\quad + P_{n,n^-}\hat{n}_t\hat{n}_{t-1} + P_{n,a}\hat{n}_t\hat{a}_t + P_{n,p}\hat{n}_t\hat{\theta}_{p,t} + P_{y,a}\hat{y}_t\hat{a}_t + P_{y,p}\hat{y}_t\hat{\theta}_{p,t}\end{aligned}$$

The correct LQ system is then given by:

$$\max_{\{\pi_t, i_t, n_t, y_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t$$

$$\begin{aligned} s.t. \quad & \pi_t = \beta E_t \pi_{t+1} + \kappa_p \left(\varnothing_1 \hat{y}_t - (1+\phi)\,\hat{a}_t - (\theta_1 \hat{n}_t + \theta_2 \hat{n}_{t-1})\right) + \theta_{p,t} \\ & \hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\varnothing_1 - \phi}\left(i_t - E_t \pi_{t+1}\right) \\ & \qquad - \frac{1}{\varnothing_1 - \phi}\left[(\theta_1 - \phi)\left(E_t \hat{n}_{t+1} - \hat{n}_t\right) + \theta_2\left(\hat{n}_t - \hat{n}_{t-1}\right)\right] \\ & \hat{y}_t = \frac{1}{(1+\varnothing_1)}\left[\gamma_1 E_t \hat{n}_{t+1} + \gamma_2 \hat{n}_t + \gamma_3 \hat{n}_{t-1} + (1+\phi)\,\hat{a}_t + \vartheta^{\kappa} E_t\left(i_t - \pi_{t+1}\right)\right] \end{aligned}$$

Search and matching model: optimal targeting tule

With the targeting rule given by:

$$
\begin{aligned}
0 \quad = \quad & \varpi_1 \hat{n}_t + \varpi_2 \hat{n}_{t-1} + \varpi_3 \hat{n}_{t+1} + \varpi_4 \hat{y}_t + \varpi_5 \hat{y}_{t+1} + \varpi_6 \hat{a}_t + \varpi_7 \hat{\theta}_{p,t} \\
& + \varpi_8 \pi_t + \varpi_9 \pi_{t+1} + \varpi_{10} \hat{P}_{t-1} + \varpi_{11} \hat{y}_t^{WA} + \varpi_{12} \hat{n}_t^{WA} \\
& + \varpi_{13} \hat{a}_t^{WA} + \varpi_{14} \hat{\theta}_{p,t}^{WA} + \varpi_{15} \hat{P}_t^{WA}
\end{aligned}
$$

where

$$
\begin{aligned}
\pi_t &= \hat{P}_t - \hat{P}_{t-1} \\
\hat{y}_t^{WA} &= \beta_\delta \hat{y}_{t-1}^{WA} + \hat{y}_t \\
\hat{n}_t^{WA} &= \beta_\delta \hat{n}_{t-1}^{WA} + \hat{n}_t \\
\hat{a}_t^{WA} &= \beta_\delta \hat{a}_{t-1}^{WA} + \hat{a}_t \\
\hat{\theta}_{p,t}^{WA} &= \beta_\delta \hat{\theta}_{p,t-1}^{WA} + \hat{\theta}_{p.t} \\
\hat{P}_t^{WA} &= \beta_\delta \hat{P}_{t-1}^{WA} + \hat{P}_t
\end{aligned}
$$

Back to presentation.

Optimal targeting rule in sticky wage model: LQ

$$\max_{\{\pi_t,\pi_t^w,x_t,i_t,\hat{w}_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(\sigma_L + \sigma_C)}{2} x_t^2 + \frac{1+\theta_p}{2\theta_p \kappa_p} \pi_t^2 + \frac{1+\theta_w}{2\theta_w \kappa_w} (\pi_t^w)^2 \right\}$$

$s.t.$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r_t^*)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p (\hat{w}_t - a_t) + \hat{\theta}_{p,t}$$

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w ((\sigma_L + \sigma_C) x_t + a_t - \hat{w}_t)$$

$$\hat{w}_t = \hat{w}_{t-1} + \pi_t^w - \pi_t$$

Optimal targeting rule in sticky wage model: F.O.C.

$$
\begin{aligned}
(\pi_t): \quad & \frac{1+\theta_p}{\theta_p \kappa_p}\pi_t + \frac{\beta^{-1}}{\sigma}\Lambda_{1,t-1} + \Lambda_{2,t-1} - \Lambda_{2,t} - \Lambda_{4,t} = 0 \\
(\pi_t^w): \quad & \frac{1+\theta_w}{\theta_w \kappa_w}\pi_t^w + \Lambda_{3,t-1} - \Lambda_{3,t} + \Lambda_{4,t} = 0 \\
(x_t): \quad & (\sigma_L + \sigma_C)\, x_t + \beta^{-1}\Lambda_{1,t-1} - \Lambda_{1,t} + \kappa_w (\sigma_L + \sigma_C)\Lambda_{3,t} \\
(i_t): \quad & \frac{1}{\sigma}\Lambda_{1,t} = 0 \\
(w_t): \quad & \kappa_p \Lambda_{2,t} - \kappa_w \Lambda_{3,t} + \beta \Lambda_{4,t+1} - \Lambda_{4,t} = 0
\end{aligned}
$$

Sticky wage model: optimal targeting rule

Eliminate the Lagrange multipliers, (and replace the output gap by output + other variables) to replace the four first order conditions by the optimal targeting rule:

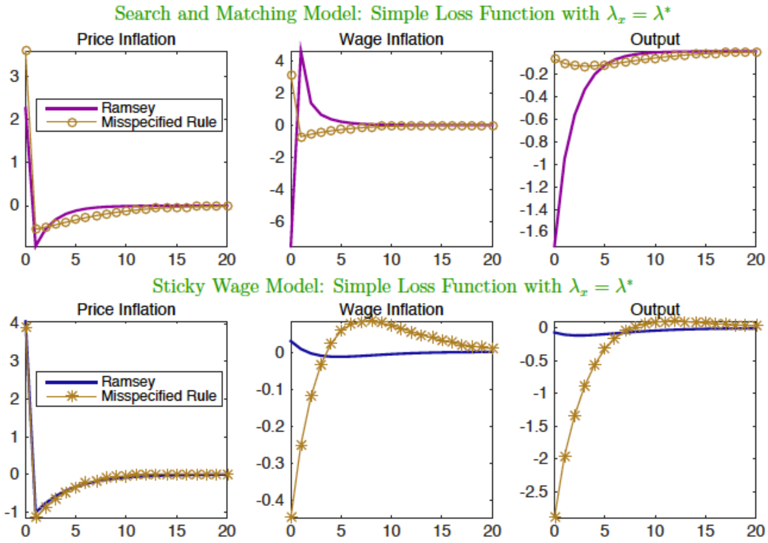
$$-\tilde{\chi}_1\pi_t = \tilde{\chi}_2\left(\pi^w_{t+1} - \pi^w_t\right) + \tilde{\chi}_3\pi^w_t + \tilde{\chi}_4\left(\pi^w_t - \pi^w_{t-1}\right)$$
$$+\tilde{\chi}_5\left(\hat{y}_{t+1} - \hat{y}_t\right) + \tilde{\chi}_6\left(\hat{y}_t - \hat{y}_{t-1}\right) + \tilde{\chi}_7\left(\hat{y}_{t-1} - \hat{y}_{t-2}\right)$$
$$+\tilde{\chi}_8\left(a_{t+1} - a_t\right) + \tilde{\chi}_9\left(a_t - a_{t-1}\right) + \tilde{\chi}_{10}\left(a_{t-1} - a_{t-2}\right)$$

Absent sticky wages, this expression reduces to:

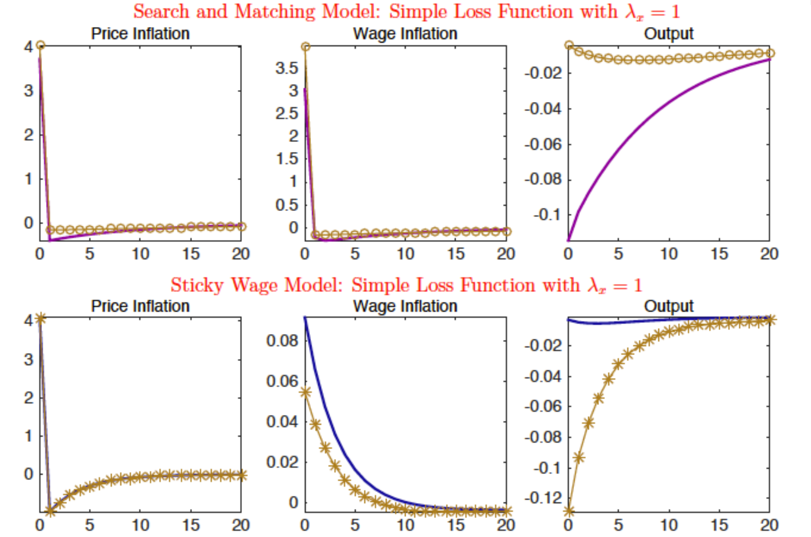
$$\pi_t = \chi_6\left(\hat{y}_t - \hat{y}_{t-1}\right) + \chi_9\left(a_t - a_{t-1}\right).$$

Back to presentation.

Asymmetric weights loss function: price markup shock



Equal weights loss function: price markup shock



Household

$$\max_{c_t^w, c_t^u, n_t, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[n_t U(c_t^w, h_t^w, 1) + (1 - n_t) U(c_t^u, h_t^u, 0) \right]$$

s.t.

$$c_t + \frac{B_{t+1}}{P_t} \leqslant \left[w_t h_t^w n_t + b^u (1 - n_t) \right] + \frac{\pi_t}{P_t} + \frac{T_t}{P_t} + \frac{R_{t-1} B_t}{P_t}$$

$$c_t = n_t c_t^w + (1 - n_t) c_t^u$$

Household

To allow for as many similarities between the two models are possible, the preferences of each member are:

$$U(c_t^i, h_t^i) = \frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \phi_0^i \frac{({h_t}^i)^{1+\phi}}{1+\phi}$$

with $i = \{u, w\}$. Back to presentation.

Nash bargaining over wages

Wage determination:

$$H_t = \xi(J_t + H_t)$$

- the value of a match to wholesale firm:

$$J_t = (mc_t a_t h_t - w_t h_t) + (1-\rho)\, E_t \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}$$

- the match surplus to household:

$$H_t = \left(w_t h_t - b^u - \frac{\phi_0}{1+\phi} h_t^{1+\phi} n_t \frac{1}{c_t^{-\sigma}} \right) + (1-\rho)\, E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - s_{t+1}) H_{t+1}$$

Nash bargaining over wages

Determination of hours worked:

$$mc_t a_t = \phi_0 h_t{}^{\phi} c_t^{\sigma}.$$

Back to presentation.

Linear approximation: search and matching model

Linearizing the equilibrium conditions of the search and matching model and substituting out unemployment and vacancies in terms of employment:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_p \left(\o_1 \hat{y}_t - (1+\phi) \hat{a}_t - (\theta_1 \hat{n}_t + \theta_2 \hat{n}_{t-1}) \right) + \theta_{p,t}$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\o_1 - \phi} \left(i_t - E_t \pi_{t+1} \right) - \frac{1}{\o_1 - \phi} \left[(\theta_1 - \phi) \left(E_t \hat{n}_{t+1} - \hat{n}_t \right) + \theta_2 \left(\hat{n}_t - \hat{n}_{t-1} \right) \right]$$

$$\hat{y}_t = \frac{1}{(1+\o_1)} \left[\gamma_1 E_t \hat{n}_{t+1} + \gamma_2 \hat{n}_t + \gamma_3 \hat{n}_{t-1} + (1+\phi) \hat{a}_t \right] + \frac{1}{(1+\o_1)} \vartheta^{\kappa} E_t \left(i_t - \pi_{t+1} \right)$$

The last equation reduces to $\hat{y}_t = \hat{n}_t + \hat{a}_t$ if the labor supply is inelastic.

Back to presentation.