

Discussion Of "Optimal Monetary Policy and Term Structure in a Continuous-Time DSGE Model"

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Research Background

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- The Ang-Piazzesi (2003) Approach

- in a purely no-arbitrage framework, e.g. [Ang, Dong and Piazzesi \(2007\)](#), [Hoerdahl, Tristani and Vestin \(2006\)](#), [Rudebusch and Wu \(2008\)](#)
- The instantaneous interest rate is an affine function of macro variables M_t and latent yield-curve factors \mathcal{P}_t ,

$$r_t = \rho_0 + \rho'_m M_t + \rho'_p \mathcal{P}_t = \rho_0 + \rho'_z Z_t, \quad Z_t = (M'_t, \mathcal{P}'_t).$$

- The dynamics of Z_t and the market price of risks have an affine structure (Duffie and Kan, 1996; Dai and Singleton, 2001; Duffee, 2002).
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 - usually based on a DSGE model, e.g. [Doh \(2011, 2012\)](#), [Rudebusch and Swanson\(2011, 2012\)](#), [Binsbergen, Fernndez-Villaverde, Koijen and Rubio-Ramrez \(2012\)](#)
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 - No-arbitrage condition is imposed by defining the stochastic discount factor, which can be determined after households value function in equilibrium is solved.
- Most extant studies with the structural approach focus on the model's **implications on the term structure of interest rates, rather than the implications on monetary policy**

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- **Objective:** To study optimal monetary policy in a DSGE model with a quadratic cost function
- **Innovations** (relative to previous policy optimization exercises, e.g., Woodford (2003)):
 - The **continuous-time** approach yields a closed-form solution for the optimal interest rate rule

$$\begin{aligned} -\rho_m V + \min_r \left\{ & \left(\tilde{\rho}(\pi - \bar{\pi}) - \kappa_y x \right) \frac{\partial V}{\partial \pi} + \frac{1}{2} \|\sigma_\pi\|^2 \frac{\partial^2 V}{\partial \pi^2} \right. \\ & + \frac{1}{\gamma} (r - \tilde{r} - (\pi - \bar{\pi})) \frac{\partial V}{\partial x} + \frac{1}{2} \|\sigma_x\|^2 \frac{\partial^2 V}{\partial x^2} + \sigma_x \cdot \sigma_\pi \frac{\partial^2 V}{\partial \pi \partial x} \\ & \left. + \frac{1}{2} \left((\pi - \bar{\pi})^2 + \alpha_x x^2 + \alpha_r r^2 \right) \right\} = 0. \end{aligned} \quad (26)$$

- There is no higher-order terms: $(dZ_t)^n = 0, n \geq 3$ and $dZ_t dt = 0$.
- A standard backward parabolic PDE.

A Summary of This Paper (cont'd)

- **Innovations** (cont'd):
 - Use of **Treasury yields** to assess optimality of monetary policy over the 1952-2007 period
 - Allow the coefficients of the interest rate rule and the dynamics of state variables to be **regime dependent**

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- **Findings:**

- There are **two distinct systematic monetary policy regimes** in the past.
- One is **not optimal** while the other is **near-optimal with a large monetary discount rate**.
- **None of the two policy regimes** can achieve absolute macroeconomic stability.

On the Yield Curve Fitting of the Proposed Model

- Does the two-factor affine model capture the observed yields well?
 - Estimates of standard deviation of measurement error σ_m are large

$$Y_t^o(\tau_m) = Y_t(\tau_m) + \epsilon_{m,t}, \quad \epsilon_{m,t} \sim N(0, \sigma_m)$$

Parameter	Single Regime		Two Regimes	
$\sigma_{m=1, \dots, 6}$	0.0217	(0.0011)	0.0134	(0.0007)
	0.0213	(0.0010)	0.0122	(0.0006)
	0.0214	(0.0010)	0.0115	(0.0006)
	0.0213	(0.0010)	0.0116	(0.0006)
	0.0215	(0.0010)	0.0120	(0.0006)
	0.0215	(0.0010)	0.0122	(0.0006)

- A three-factor Gaussian TSM with the Joslin-Singleton-Zhu (2011) canonical form implies that $\sigma_m < 15\text{bps}$ for all maturities.
- Fitting errors are likely to be cross-sectionally and serially correlated.

Is R-squared a good measure for the model's empirical performance?

- Observed Yields = $\gamma_0 + \gamma_1$ Model Yields + error

R^2 s from Regressions of Observed Treasury Yields

Maturity (yr)	Linear	Trend	Model Yields	
	Trend	Inflation	Single Regime	Two Regimes
1	0.325	0.714	0.455	0.817
2	0.311	0.770	0.429	0.836
3	0.303	0.798	0.404	0.829
4	0.286	0.826	0.389	0.819
5	0.271	0.841	0.381	0.807

- R^2 s from regressions of yield levels are “cheap”.
- More convincing to run regressions of yield changes

- Solution #1: Incorporating latent yield-curve factors

$$r_t = \rho_0 + \rho_g g_t + \rho_\pi \pi_t + \rho'_p \mathcal{P}_t.$$

- Drawbacks
 - Joslin, Le and Singleton (2013) show that this type of models is rotation-invariant: different choices of \mathcal{P}_t give rise to theoretically equivalent model representations but different coefficients (ρ_g, ρ_π) .
 - Without imposing **additional economic structure** on \mathcal{P}_t , (ρ_g, ρ_π) are not meaningfully interpretable as the reaction coefficients of a central bank.

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- Solution #2: Making \mathcal{P}_t endogenous
 - E.g., Ang, Dong and Piazzesi (2007) interpret their \mathcal{P}_t as a policy shock.
 - Drawback: Additional theoretical effort to include it into the monetary loss function.

Possible Solutions (cont'd)

- Solution #3: Assuming that macroeconomic variables are measured with error, e.g.

$$\pi_t^o = \pi_t + \epsilon_{\pi,t}, \quad \epsilon_{\pi,t} \sim N(0, \sigma_\pi)$$

- The likelihood function largely gives up on fitting the observed macro factors in favor of more accurate pricing of bonds (Joslin, Le and Singleton, 2013).
- If anything, it seems more natural to assume that the macro variables are observed with errors and yields are observed perfectly.
- Drawback: Estimate of σ_π would be large.

- Solution #4: Accommodating time variations in term premia

$$Y_t^{(\tau)} = \frac{1}{\tau} \sum_{i=0}^{\tau-1} E_t(Y_{t+i}^{(1)}) + \frac{1}{\tau} \sum_{i=1}^{\tau-1} E_t(rx_{t+i}^{(\tau-i+1)})$$

- The second term is **constant** within each regime.
 - $E_t(rx_{t+1}^{(\tau)}) = B(\tau)\Sigma\Lambda_t$
 - $\Lambda_t \equiv \Lambda = \sigma_p + \gamma\sigma_g$
- Drawback: Need to modify the household's utility function

- Point forecasting v.s. [density forecasting](#)
 - The latter is important for the pricing of interest rate derivatives.
 - Hong, Li and Zhao (2004) show that DTSMs produce better density forecasts than the random walk.
- Need to reinvestigate the claim that the JSZ estimator does not take into account the time-series properties of the state variables.
 - The state variables used in \mathbb{P} -measure estimation, $\mathcal{P}_t = WY_t$, could be selective zero yields.
 - If \mathcal{P}_t are 0.5-, 2-, and 10-yr zero yields as is the “RY” case in JSZ,
 - Maximizing the \mathbb{P} -measure likelihood is equivalent to minimizing one-step-ahead yield forecasting errors

$$L(K_{0\mathcal{P}}^{\mathcal{P}}, K_{1\mathcal{P}}^{\mathcal{P}}) \propto -e^{\sum_{t=1}^T \|\Sigma_{\mathcal{P}}^{-1}(\mathcal{P}_{t+1}^o - E_t(\mathcal{P}_{t+1}|K_{0\mathcal{P}}^{\mathcal{P}}, K_{1\mathcal{P}}^{\mathcal{P}}))\|^2}$$

Conclusion

- Interesting paper that matters a lot especially to macroeconomists and policy makers
- Raise some intriguing questions that may lead to many follow up papers
- If the combination of macroeconomics and term structure holds promise, it also raises challenges.