Optimal Sovereign Defaults in the Presence of Financial Frictions *

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Abstract

Why are sovereign defaults often accompanied by significant declines in economic aggregates, and what determines the decisions of the governments to default on their debts? To answer these questions, I develop a model of sovereign default in a production economy with financial frictions. A key feature of the model is that the amount of working capital loans that banks can extend to firms is limited by banks' assets, which include both physical capital and government bonds. Defaulting on debt reduces banks' assets and thus their lending, thereby inducing firms to cut back on production. The government trades off these costs of lowered output against the distortions from taxes needed to repay the debt. I calibrate the model separately to Argentine and Italian data. In the model, Argentina sustains a lower debt level with high default rate while Italy sustains a higher debt level with negligible default rate. The model also successfully captures the declines in output and investment associated with defaults at observed default frequencies.

Keywords: Sovereign default; Financial friction; Optimal fiscal policy; Time-consistency problem; Endogenous cost of default.

JEL Codes: E32, E44, E62, F34

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1 Introduction

Sovereign defaults are often associated with significant declines in economic activity in defaulting countries. For example, Levy-Yeyati and Panizza (2011), Reinhart and Rogoff (2011), Mendoza and Yue (2012), and Tomz and Wright (2013) all find that GDP falls at least 5% below trend during default episodes. Another interesting but less well-known fact is that sovereign governments not only borrow a large fraction of their debts from domestic residents and institutions, but also default on them. For instance, Reinhart and Rogoff (2011) document that there are many cases of domestic sovereign defaults and their macroeconomic effects are as significant as those of external defaults. They also show that domestic public debt accounts for a large fraction of total government debt in many countries. These observations raise two interesting and puzzling questions. Why are defaults often accompanied by recessions, and why do the governments default on their own citizens?

I attempt to answer these questions by developing a model of sovereign default in a production economy with capital and financial frictions. The essential idea is that banks are exposed to government debt and a sovereign default decreases the values of banks' assets. This disruption forces banks to reduce credit to firms, which weakens the economy's capacity to produce. Nevertheless, a benevolent government may still choose to default occasionally, despite its negative effects on economic activity. The reason is that a default reduces the tax burden of repaying debt.

I focus on a closed economy to capture the idea that the holders of government bonds are domestic residents and the government still defaults on them. In the model economy, firms have to finance working capital in advance before production takes place. The amount of working capital is positively related to the scale of production. Banks intermediates these loans from households to firms. Due to limited enforcement, the amount of loans is limited by banks' assets, which include government bonds and physical capital. A sovereign default thus directly reduces banks' assets and tightens their enforcement constraints, which increases financing costs for firms. With higher financing costs, firms have to reduce their demand for labor and cut back on production. This leads to an output decline.

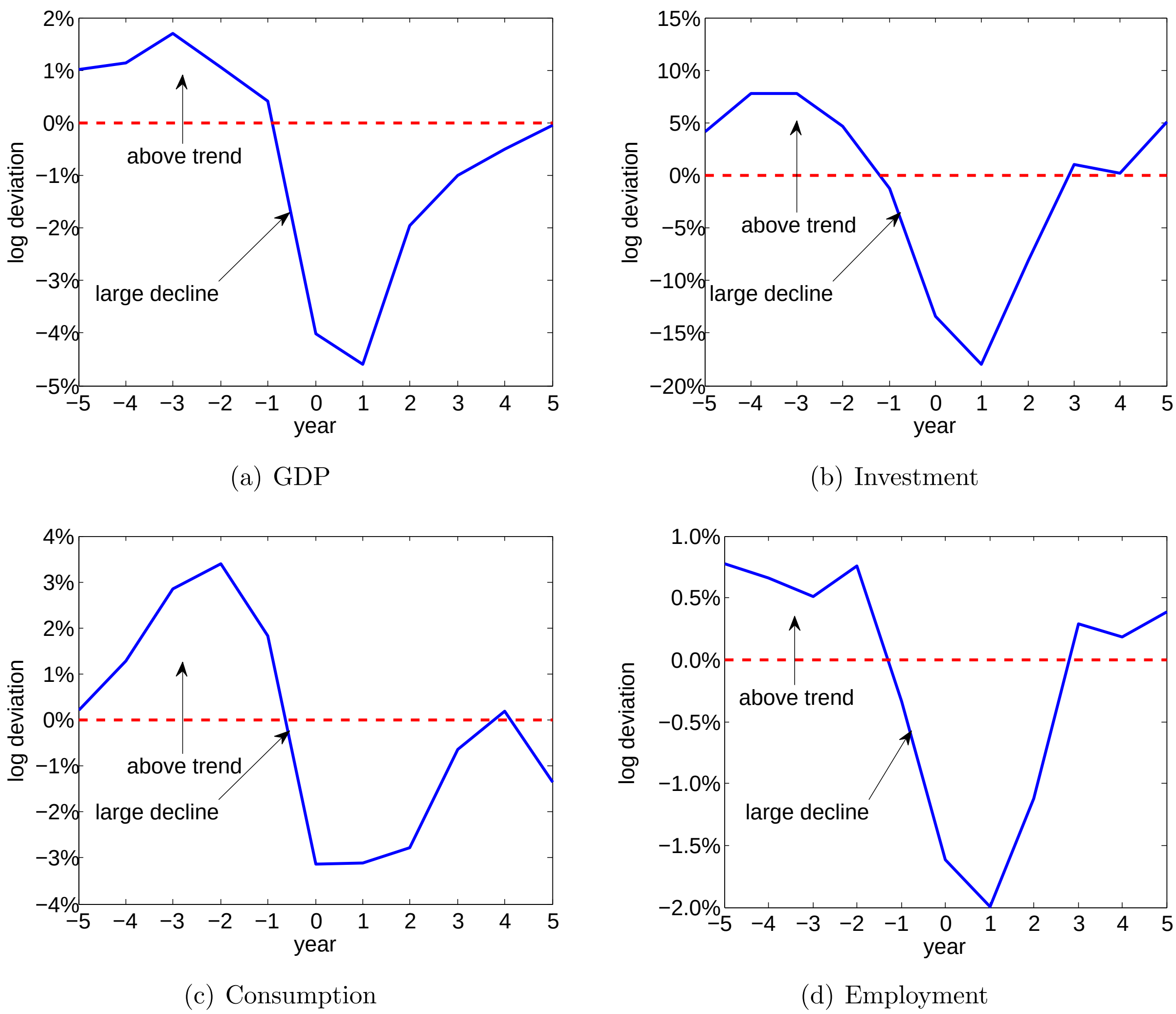
Despite the adverse impact, defaults do occur occasionally. In the model, the government needs to finance exogenous spending. In order to do so, the government has three instruments: raising new debt, levying taxes and/or defaulting on outstanding debt. As taxes are distortionary, the government may find it optimal to default when the level of debt is already high. The benefits of default are reduced taxes, which imply less distortion on the economy. The costs of default are reduced credit and decreased output. The government balances these tradeoffs, and makes default decisions when the costs of distorting taxes of repaying the debt outweighs the costs of output loss associated with default.

This paper therefore provides a transmission mechanism between default and production, which generates endogenous costs to sustain risky debt. Since the seminal work of Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), international macroeconomists have argued that the key to understanding sovereign debt is understanding the incentives of the government to repay its debt. That is, default must be associated with some punishment. The canonical way of modeling punishment for default is to simply assume that output decreases exogenously following a default. This approach is adopted by many researchers including Cole and Kehoe (2000), Aguiar and Gopinath (2006), and Arellano (2008). This paper introduces default costs that arise endogenously in the model by endowing government debt with a role of providing liquidity in private credit market and facilitating production.

By modeling the interactions between debt and production, this paper provides a new dimension to understand the government's incentives to repay or to default. In the model, the government's temptation to default is the strongest when the economy encounters a large negative TFP shock after a sequence of positive shocks. The reason is that in response to a sequence of positive TFP shocks, investment increases and the economy builds up a high level of capital stock. A large amount of capital implies that banks have more assets, which diminishes the importance of government bonds to satisfy the enforcement constraint, so the cost of defaulting is lower. At the same time, when the shock to TFP is large and negative, repaying debt is more burdensome because it requires the government to charge a higher tax rate. Hence, defaulting on debt at this state reduces tax distortions by a greater amount relative to when the state of the economy is good. Lower default costs and higher default benefits induce the government to default when the capital stock is relatively high and TFP is relatively low. In a stationary and stochastic environment, having a sequence of positive shocks followed by a large negative shock is a low probability event, which explains why defaults occur infrequently.

Incorporating capital accumulation in the model thus leads to important implications. In the data, while defaults are often characterized by decreases in output, the associated declines in investment are generally much larger. The percentage decrease in investment is usually three to four times that of the decline in GDP. In the model, the government has a strong incentive to default when the economy has a high capital stock and receives a low TFP shock. When the capital stock is high and TFP is low, the economy-wide return to capital is very low, so investment decreases. At the same time, current consumption is low relative to future consumption, so the stochastic discount factor falls. This implies that firms discount the future at a higher rate, which further decreases investment. This reduction in investment has adverse and persistent impact on future output.

Figure 1 provides some evidence for the key implications of the model. Using data from 23 default events between 1980 and 2005, I plot in Figure 1 the log deviations of GDP, investment, consumption and employment, centered around the year of default.[1] On the x-axis, 0 is the year of default, $-t$ and t are t years before and after respectively. Clearly, all variables decrease sharply around the time of default. While GDP falls to about 5% below trend, investment decreases to almost 20% below trend. This huge decline in investment is an under-appreciated feature in the default literature because the existing literature assumes either an endowment economy or a production economy without capital. The dynamics of investment and its interactions with default highlight the important role that capital accumulation plays both in the data and in the model.



(a) GDP

(b) Investment

(c) Consumption

(d) Employment

Figure 1: Macroeconomic Dynamics around Default

[1] All variables are annual series and HP-filtered. For descriptions of the data and the list of countries, refer to Appendix A.

The inclusion of capital accumulation and the presence of enforcement constraints are the keys to generating the default pattern highlighted in the paper. That is, the government is more likely to default when the economy experiences a sequence of positive shocks followed by a large negative shock. This seems to be consistent with the experiences of many countries that have defaulted. Many of these countries have been growing above trend for some time before they encounter a sudden downturn and crises occur. As illustrated in Figure 1, GDP, investment, consumption and employment are all above trend until a year before the default crisis. This paper is able to account for this interesting pattern. The economy increases investment and builds up capital stock in periods of expansion. What triggers default then is a big fall in productivity. At this time, the default costs become smaller, because the high level of capital stock built up in previous periods implies there is relatively sufficient assets in the economy to withstand the negative shock.

In the quantitative section of the paper, I calibrate the model separately to the Argentine and the Italian economy. Argentina is an example of an emerging economy that has a history of default, while Italy is an example of a developed country that is exposed to default risk in the European debt crisis. Calibrating to Argentina provides a benchmark to test the model's performance in matching data, while calibrating to Italy allows me to answer normative questions about default risks in the debt crisis in Europe. The main result of the calibration is that the model is consistent with both types of economies.

The model generates enough government commitment to sustain observed levels of domestic debt and default frequencies in both countries. Specifically, in the simulations, Argentina sustains a low domestic debt to GDP ratio of 27% with a high annual default rate of 0.78%, while Italy sustains a high debt ratio of 59% with an almost zero default rate.[2] In the model, as in the data, Argentina defaults much more frequently than Italy, because its TFP process is much more volatile. As default is a response to alleviate debt burdens when economic conditions worsen, a very volatile TFP implies that the government will use this policy more often. With higher default rate, less debt is sustained in the Argentine economy. Furthermore, the model also successfully captures the declines in output and investment associated with defaults. In the simulations, output drops around 11% in Argentine defaults, which is a close match to the data. In a counterfactual analysis, the model shows that if Italy defaults, output decreases around 6%.

To further understand the differences in default between Argentina and Italy, I conduct counterfactual analysis on the Argentine economy using parameters from the Italian calibration. Two important differences between the Argentine and the Italian model economies are

[2]The default rate here is the domestic debt default rate, which is lower than the external debt default rate that has a typical value in the range of $2.5\% - 3\%$.

the financial friction (the enforcement constraint) and the TFP process parameters. I change these two sets of parameters one at a time. In the first exercise, if Argentina has the level of financial frictions as that of Italy, its default probability decreases to 0.27% and output loss increases slightly to 11.8%. The reason is that Italy has a closer connection between the financial sector and the real sector. This interdependence increases the potential default cost, which in turn induces the government to default less. In the second exercise, if Argentina has the Italian TFP process, its default probability and output loss reduce to 0.24% and 4.6% respectively. This is because Italy has a much less volatile TFP, which decreases the likelihood of default and lowers output variability.

Many empirical papers on sovereign defaults lend support to the idea that costs of default operate primarily through its impact on the domestic economy via the financial sector, rather than the external channels. Panizza, Sturzenegger and Zettelmeyer (2009) find limited empirical support for default costs based on market exclusion and external sanction, and more support for default costs brought on the domestic economy. Gennaioli, Martin and Rossi (2014a) document that banks' holdings of government bonds correlate negatively with their lending to the private sector during defaults. For the European debt crisis, Acharya et al. (2014) show that the exposure of banks to sovereign bonds and the resulting credit crunch contribute to the severe economic downturn. All these findings suggest that government bonds play a role in the domestic private sector and default disrupts its function.

An important paper that introduces production into default model is Mendoza and Yue (2012). In their model, a fraction of intermediate goods are produced by foreign markets. When the government defaults, firms lose access to trade credits and those foreign intermediate goods have to be replaced by less efficient domestic intermediate goods. This source of inefficiency decreases output. My paper focuses on a different aspect by incorporating a banking sector and analyzing the impact of the credit crunch. Furthermore, it allows for capital accumulation, which enriches the government's default decision by making it not only depend on the level of outstanding debt, but also on the level of capital.

Motivated by strong empirical evidence, Gennaioli, Martin and Rossi (2014b), Sosa-Padilla (2014), and Perez (2014) study the effects of sovereign defaults on domestic banks, and the associated output losses. While we share a similar mechanism, their models do not have capital accumulation, making them unsuitable to answer questions about business cycle dynamics. In addition, I explicitly model the behavior of the financial sector and credit market. Another related paper is Bocola (2014), who studies the interactions between firms, banks and the government. But the dynamics of government debt and default risk are both exogenous in his model, while I endogenize both in this paper.

This paper contributes to the study of domestic sovereign defaults. Reinhart and Rogoff (2011) document that there are many cases of domestic sovereign defaults, but this area has been largely unexplored in the literature. Domestic and external defaults differ in at least two important aspects. First, bondholders are domestic residents and the government values their welfare in domestic defaults, whereas bondholders are international investors and the government does not care about them in external defaults. This makes domestic defaults less likely to occur. Second, the decision to default involves different trade-offs. In external defaults, benefits of default are net transfers from foreigners to domestic residents, and costs are (exogenous) output losses. In domestic defaults, there are no net transfers. The trade-off emphasized in this paper is between distortionary taxation and distortions induced by financial frictions.

Pouzo and Presno (2015) study domestic default and focus on a similar trade-off as in this paper. However, their model features exogenous default cost, and assume the government has commitment to tax policy but not to repayment policy. This paper assumes the government has no commitment to both tax and repayment policies. D'Erasmo and Mendoza (2013) also study domestic default, but in an endowment economy and argue that the government defaults in order to redistribute wealth among residents.

The remainder of the paper is as follows. Section 2 describes the model and characterizes the private sector's optimality conditions. Section 3 describes the government's problem and defines the Markov perfect equilibrium. Section 4 presents the calibration strategy. Section 5 shows the quantitative results and robustness analysis. Section 6 compares and contrasts the model economies of Argentina and Italy. Section 7 provides concluding remarks. Descriptions of data, derivations, and other materials are in the Appendix.

2 Model

In this section, I formulate a model of domestic default in a business cycle economy with financial frictions. The modeling of financial frictions is similar to that in Jermann and Quadrini (2012). I introduce in this setting risky government bonds. I also introduce a benevolent government who lacks commitment.

Time is discrete and infinite. The economy is populated by households, firms, banks and the government. To finance exogenous spending, the government taxes households' labor income and issues bonds, but it can default on its debt. The government chooses its tax, debt and default policies to maximize the welfare of households, who are the ultimate owners of both banks and firms.

In order to produce, firms have to finance some working capital loan, which is related to the scale of production. Banks intermediates funds from households to firms. But due to limited enforcement, the amount of borrowing and lending is limited by banks' assets, which are comprised of both physical capital and government bonds. A default reduces banks' assets, and thus reduces the amount of working capital loan banks can lend to firms, thereby forcing firms to decrease production. The presence of this endogenous cost helps to explain the sustainability of public debt and the incentives to default.

In the remainder of this section, I illustrate the timing of events, describe the agents in the economy, define the competitive equilibrium, and characterize the optimality conditions.

2.1 Timing

The timing of events within a period is illustrated in Figure 2 and it proceeds as follows:

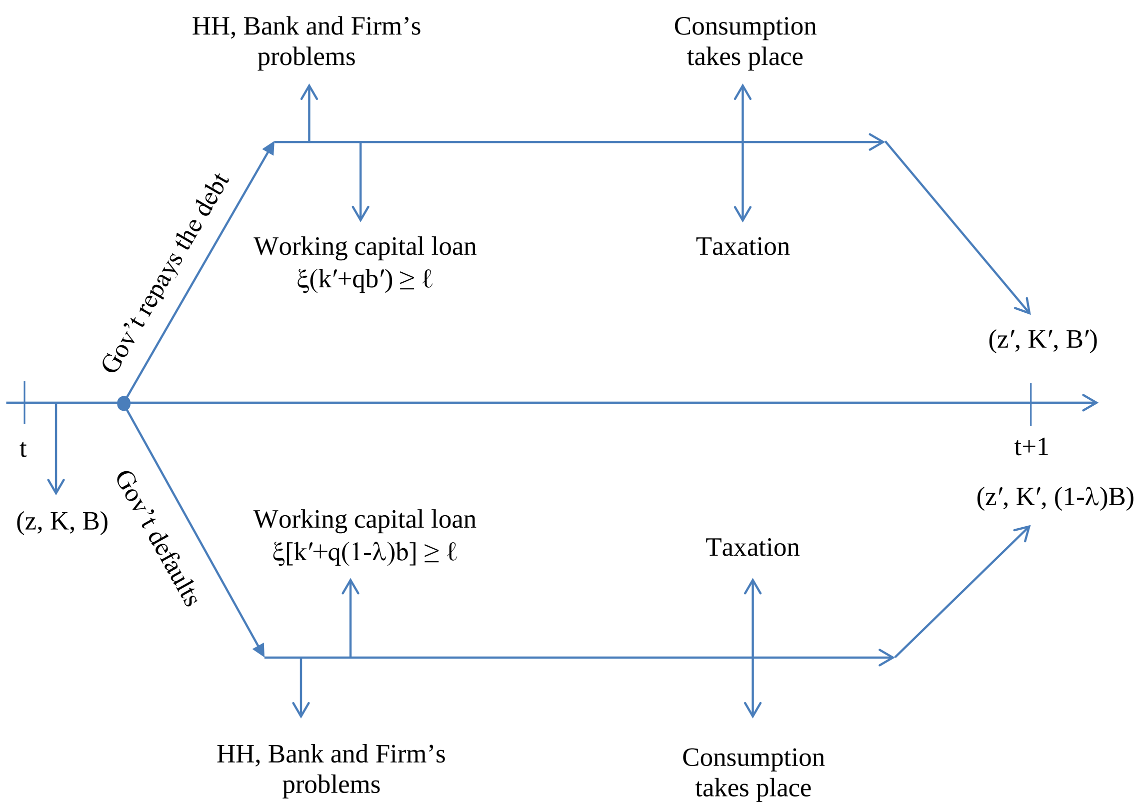


Figure 2: Timing of Events

1. At the start of period t, the exogenous productivity shock z_t is observed.

2. The aggregate state variables are TFP, capital stock and bond holdings (z_t, K_t, B_t).

3. The government makes default decision: $D_t \in \{0, 1\}$.

- If the government chooses to repay ($D_t = 0$), the following happens:
  - The government pays back its outstanding debt, issues new debt, and announces tax plan.
- If the government chooses to default ($D_t = 1$), the following happens:
  - The government writes off a fraction λ of its outstanding debt and announces tax plan.
  - The government cannot issue new debt in this period and rolls over the non-defaulted debt.

4. Firms decide how much capital to rent and labor to hire. Firms borrow to meet the working capital requirement.

5. Banks decide how much bonds to purchase, investment to make and dividends to issue. Banks intermediate households' deposits to firms.

   - The amount of working capital loan is subject to an enforcement constraint, which I will describe in detail later.

6. Households decide how much labor to supply, and how much to consume.

7. All markets clear. Firms repay working capital loans to banks. Banks repay deposits to households. Households pay taxes and consume.

8. Period t ends. The new level of capital stock is K_{t+1}. The new level of government debt is B_{t+1} if the government repays or $(1-\lambda)B_t$ if the government defaults.

2.2 Households

There is a continuum of identical households. Households value consumption c_t and dislike labor n_t according to the flow utility $U(c_t, n_t)$, and they discount future at the rate $\beta \in (0,1)$. Households supply labor to firms. Households also trade equity shares of banks and receive dividends from holding equities. Takings taxes, prices and dividends as given, a household maximizes the expected lifetime utility,

$$\max_{c_t, n_t, e_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t), \tag{1}$$

subject to the budget constraint,

$$c_t + p_t e_{t+1} = (1-\tau_t) w_t n_t + (p_t + d_t) e_t,$$

where w_t is the wage rate, τ_t is the tax rate on labor income, e_t are the equity shares, p_t is the market price of shares, and d_t are the dividends received from banks.

2.3 Firms

There is a continuum of homogeneous firms, which are owned by banks. Firms operate a constant returns to scale production technology, $F(z_t, k_t, n_t) = z_t k_t^{\alpha} n_t^{1-\alpha}$. The variable z_t is the stochastic technology shock common to all firms, k_t is the capital input, and n_t is the labor input. Firms rent capital from banks and hire labor from households. The rental rate of capital r_t and the wage rate w_t are both determined in competitive markets.

One friction in the model is that firms face a working capital constraint. Firms are required to make payments for capital and labor before the realization of revenues.[3] In order to do so, firms take intraperiod working capital loans ℓ_t^f from banks. After output is realized, firms use revenues to repay the loans. There are no interests on intraperiod loans. A firm maximizes its profit,

$$\begin{aligned} \Pi_t &= \max_{k_t, n_t, \ell_t} F(z_t, k_t, n_t) - r_t k_t - w_t n_t + \ell_t^f - \ell_t^f, \qquad (2) \\ \text{subject to} \quad & \ell_t^f = r_t k_t + w_t n_t. \end{aligned}$$

2.4 Banks

There is a continuum of homogeneous banks. Banks are the financial intermediaries that transfer funds between households and firms. As banks owns firms, banks receive firms' profits. Banks also accumulate their own assets, in the form of physical capital and government bonds.

Capital k_t is chosen at time $t-1$ and thus predetermined at time t. Capital evolves according to $k_{t+1} = (1-\delta)k_t + i_t$, where δ is the rate of depreciation and i_t is investment. Banks rent out capital to firms with the rate of return r_t.

Government bond b_t is a risky one-period security, which pays one unit of consumption in the following period if the government repays and $1-\lambda$ unit of consumption if the government defaults. λ is the haircut of the debt should the government defaults.

As described earlier, within a period, firms need to raise funds with intraperiod loans, ℓ_t^f, from banks to finance working capital. In order to make working capital loans to firms, banks take intraperiod deposits ℓ_t from households. Banks thus act as financial intermediaries

[3] The assumption that the working capital loan is used to pay for capital and labor inputs are not crucial. The results in the paper carry through as long as the amount of working capital loan is positively related to the scale of production. I will explain this point in greater details in the section on calibration.

between households and firms. Deposits and loans are both made at the beginning of the period, and repaid at the end of the period with no interests.[4] I assume that banks are able to raise enough deposits from households, as long as banks repay these deposits, thus $\ell_t = \ell_t^f$.[5]

A bank starts off a period with capital stock k_t and government bonds b_t. For the convenience of illustration, I assume for the time being that the government chooses not to default. A bank's budget constraint is

$$d_t + k_{t+1} + q_t b_{t+1} = (1-\delta)k_t + r_t k_t + b_t + \Pi_t + (\ell_t^f - \ell_t^f) + (\ell_t - \ell_t),$$

where d_t is the dividend made to households, q_t is the price of bonds, and Π_t are firms' profits. As the intraperiod deposits ℓ_t and loans ℓ_t^f are all canceled out, we can simply write the bank's budget constraint as

$$d_t + k_{t+1} + q_t b_{t+1} = (1-\delta)k_t + r_t k_t + b_t + \Pi_t.$$

In the model economy, there is limited enforcement of debt contracts. Banks can choose not to repay households' deposits. If a bank does not repay, households will liquidate the bank's assets, which are physical capital k_{t+1} and government bonds $q_t b_{t+1}$. Because government bonds are priced at q_t in period t, the value of bonds is worth $q_t b_{t+1}$. Suppose that households can recover a fraction ξ of the bank's net worth $k_{t+1} + q_t b_{t+1}$.[6] In Appendix B, there is a description of the renegotiation process between the bank and the lenders if the bank does not repay. Based on the outcomes of the renegotiation, we can derive the bank's enforcement constraint,

$$\xi(k_{t+1} + q_t b_{t+1}) \geq \ell_t. \tag{3}$$

The enforcement constraint implies that the amount of intraperiod loans cannot be higher than the fraction ξ of banks' assets $k_{t+1} + q_t b_{t+1}$. However, higher capital stock and/or higher holdings of government bonds relax the enforcement constraint.[7] In other words, by facilitating the reallocation of resources to enhance production, capital and government bonds help to overcome the financial friction in the economy.

[4]The assumption that intraperiod borrowing and lending have no interests simplifies computation, but the analysis carries through without this assumption.

[5]The rationale for this assumption is that in many developing countries, residents only have a limited number of saving instruments, and banks generally have monopoly power in the savings market. This is a form of financial repression.

[6]An equivalent way is to assume that with probability ξ, households can recover the full value $k_{t+1}+q_t b_{t+1}$. With probability $1-\xi$, the recovery value is zero.

[7]Dang, Gorton and Holmstrom (2012) provide some micro foundation and explanations that government debt is an optimal form of collateral and liquidity.

There are at least two different but equivalent ways to interpret the enforcement constraint. If we think of $k_{t+1}+q_t b_{t+1}$ as the collateral, then we can call equation (3) the collateral constraint where a bank's borrowing is limited by the value of its collateral. Another way is to rewrite equation (3) as

$$\frac{k_{t+1}+q_t b_{t+1}-\ell_t}{k_{t+1}+q_t b_{t+1}} \geq 1-\xi.$$

This is the equity to capital constraint where a bank's equity has to be more than a certain fraction of its assets. Nevertheless, all interpretations imply that the capacity of a bank to transfer funds is bounded by the value of its assets.

If the government chooses to default, then a bank's budget constraint and enforcement constraint are

$$\begin{aligned} d_t + k_{t+1} + q_t b_{t+1} &= (1-\delta)k_t + r_t k_t + (1-\lambda)b_t + \Pi_t, \\ \xi(k_{t+1} + q_t b_{t+1}) &\geq \ell_t, \end{aligned}$$

where λ is the haircut on the outstanding debt. I assume that the government cannot raise new debt in the period in which it defaults. However, the fraction of debt that is not defaulted can still be priced because agents can buy and sell these bonds in the secondary market.[8]

2.4.1 Recursive Formulation of a Bank's Problem

The individual states of a bank are the capital stock k and government bonds b. The aggregate states, to be specified later, are denoted by S. Given the individual and aggregate states, let $W(k,b;S)$ be the value of the bank. The bank's problem is

$$\begin{aligned} W(k,b;S) &= \max_{d,\ell,k',b'} d + \beta \mathbb{E}\left[\frac{U_c(c',n')}{U_c(c,n)} W(k',b';S')\right], \qquad (4) \\ &\text{subject to} \\ d + k' + qb' &= (1-\delta)k + rk + (1-D\lambda)b + \Pi, \\ \xi(k'+qb') &\geq \ell. \end{aligned}$$

where $D \in \{0,1\}$ is an indicator function for whether the government repays or defaults on its debt. Since banks are owned by households, $\beta\frac{U_c(c',n')}{U_c(c,n)}$ is a bank's stochastic discount factor. An individual bank takes as given the stochastic discount factor, the return to capital and the price of bonds, which are all determined in the general equilibrium.

[8] Broner, Martin and Ventura (2010) show that sovereign debt can be sustained without any default penalties due to the presence of secondary markets.

2.5 The Government

The government is a benevolent social planner, using its policy instruments to maximize the welfare of households.[9] However, it has no commitment to its debt and tax policies. Every period, the government decides if it is going to default on its debt, how much to tax τ, and how much new debt B' to issue if it chooses not to default. Tax is on labor income while debt is one-period securities.[10] In addition, the government has to finance some public spending. I assume public spending as a fraction of GDP is constant at a ratio of g.[11] The government finances its spending and pays back its outstanding debt by levying taxes on households' labor income and by issuing new government bonds to banks.

Government bonds are risky because the government can default $D \in \{0, 1\}$ in every period, where $D = 0$ means repayment and $D = 1$ means default. When the government defaults, it writes off a fraction $\lambda \in [0, 1]$ of its outstanding debt. The parameter λ is the "haircut" that the government imposes on bondholders when it defaults. Let B be the amount of the outstanding debt. If the government chooses to repay its debt, it can issue new bond B' at price q. If the government chooses to default on its debt, it cannot issue new debt for the current period and rolls over the non-defaulted portion of the debt. This means that the non-defaulted debt can still be traded. Denoting by Y the aggregate output in the economy, the government's budget constraint is

$$\begin{cases} gY + B = qB' + \tau wn & \text{if repays,} \\ gY + (1-\lambda)B = q(1-\lambda)B + \tau wn & \text{if defaults.} \end{cases}$$

In the sovereign default literature, there are two common assumptions. The first is that the government defaults on all of its debt. However, partial defaults are prevalent phenomenon in reality. For example, Sturzenegger and Zettelmeyer (2008) find that there is a wide variation in haircuts, from 13% to 73%. Cruces and Trebesch (2013) find similar pattern, with an average haircut of 37%. Benjamin and Wright (2013), and Arellano, Mateos-Planas and Rios-Rull (2013) both find an average haircut around 50%. These facts guide my assumption of partial default in the model, where λ is the haircut on debt.

Another common assumption in the literature is that the government, if defaults, is

[9] Since households own banks that in turn own firms, households are the ultimate owners of all resources in the economy.

[10] Due to the already complicated computation of the model, I abstract from capital income tax and long-term debt, even though both can generate interesting dynamics. For interactions between optimal labor and capital income taxes, see Chari and Kehoe (1999) and Conesa, Kitao and Krueger (2009). For risky long-term debt, see Chatterjee and Eyigungor (2012) and Arellano and Ramanarayanan (2012).

[11] Public spending g can be endogenized by assuming a utility function that values public spending. It can also be made stochastic at the cost of adding one more state variable.

excluded from the credit market for an exogenous period of time. However, a vast empirical work has shown that governments in defaults get back to the credit market fairly quickly. Arellano, Mateos-Planas and Rios-Rull (2013) document that countries continue to have access to credit markets during defaults. Gelos, Sahay and Sandleris (2011) report that credit market exclusion is short-lived and has declined to two years since the 1990s. For domestic credit market access, the length of exclusion is even shorter. I therefore include limited market exclusion, where the government cannot issue debt only in the period in which it defaults. Since the time period is annual in the model, it means the credit market exclusion lasts for one year.

2.6 Competitive Equilibrium

In most of the models of sovereign default, there are no individual agents, and the government is the only player in the domestic economy. While in many models in the financial friction literature, there are either no government or government policies are exogenous. In contrast, this paper features both individual agents and the government as active players in the economy. Individuals take as given expected government policies to make their intratemporal and intertemporal optimizations. The government takes as given individual policies to maximize the welfare of the residents.

Given this structure and the time inconsistency problem, I focus on a Markov perfect equilibrium in which policies are functions of payoff relevant state variables. In particular, I define the equilibrium in two steps. First, I define a competitive equilibrium given a government policy. Then, in Section 3, I define a Markov perfect equilibrium where the government policy is time consistent.

Denote the aggregate states by $S = \{z, K, B\}$, where z the productivity shock, K is the aggregate capital stock and B is the level of government debt. Denote government polices by $\pi(S) = \{D(S), B'(S), \tau(S)\}$. Let the superscript f stand for firms.

Definition 1: Given government policies $\pi(S)$, a recursive competitive equilibrium is defined as a set of functions for (1) household's consumption $c(S, \pi(S))$, labor supply $n(S, \pi(S))$, and equity share $e'(S, \pi(S))$; (2) firm's capital input $k^f(S, \pi(S))$, labor demand $n^f(S, \pi(S))$, and working capital loan $\ell^f(S, \pi(S))$; (3) bank's dividend $d(k, b; S, \pi(S))$, capital $k'(k, b; S, \pi(S))$, bond $b'(k, b; S, \pi(S))$, and deposits $\ell(k, b; S, \pi(S))$; (4) rental return $r(S, \pi(S))$, wage rate $w(S, \pi(S))$, price of bonds $q(S, \pi(S))$, and price of equity $p(S, \pi(S))$; (5) law of motion for aggregate states $S' = \Psi(S, \pi(S))$ such that:[12]

[12]Technically, it should be $x(S)$ instead of $x(S, \pi(S))$, where x is the variable of interest. I explicitly write in this form to show the dependence of allocations and prices on government policies.

1. household's policies solve household's problem (1);
2. firm's policies solve firm's problem (2);
3. bank's policies solve bank's problem (4);
4. prices $\{r(S,\pi(S)), w(S,\pi(S)), q(S,\pi(S)), p(S,\pi(S))\}$ clear capital market, labor market, bond market, and equity market:

$$\begin{aligned} k^f(S,\pi(S)) &= K, \\ n^f(S,\pi(S)) &= n(S,\pi(S)), \\ b'(K,B;S,\pi(S)) &= B'(S), \\ e'(S,\pi(S)) &= 1; \end{aligned}$$

5. intraperiod loan market clears:

$$\ell(K,B;S,\pi(S)) = \ell^f(S,\pi(S)) = r(S,\pi(S))K + w(S,\pi(S))n(S,\pi(S));$$

6. aggregate resource constraint holds:

$$c(S,\pi(S)) + K'(S,\pi(S)) = (1-\delta)K + (1-g)zK^\alpha n(S,\pi(S))^{1-\alpha};$$

7. the law of motion $\Psi(S,\pi(S))$ is consistent with individual decisions:

$$K'(S,\pi(S)) = k'(K,B;S,\pi(S)).$$

2.7 Characterizations

Households' intratemporal and intertemporal Euler equations are

$$\frac{U_n(c,n)}{U_c(c,n)} = -(1-\tau)w, \tag{5}$$

$$pU_c(c,n) = \beta\mathbb{E}\left[(p'+d')U_c(c',n')\right]. \tag{6}$$

The first condition determines the supply of labor, and the second condition determines the purchase of equity shares.

Since banks own firms and due to the constant returns to scale production technology, we can combine banks and firms together into a single entity.[13] Substituting the firm's profit into the bank's budget constraint, we have the following maximization problem,

$$
\begin{aligned}
W(k,b;S) &= \max_{d,n,k',b'} d + \beta \mathbb{E}\left[\frac{U_c(c',n')}{U_c(c,n)} W(k',b';S')\right], \\
&\text{subject to} \\
d + k' + qb' &= (1-\delta)k + F(z,k,n) - wn + (1-D\lambda)b, && (7)\\
\xi(k'+qb') &\geq F(z,k,n). && (8)
\end{aligned}
$$

where equations (7) and (8) are the budget constraint and the enforcement constraint respectively. From now on, I will refer to this entity as the (financial-and-nonfinancial) firm.

Let μ be the Lagrange multiplier associated with the enforcement constraint, the first-order conditions for n, k', and b' are[14]

$$
\begin{aligned}
F_n(z,k,n) &= \frac{w}{1-\mu}, && (9)\\
1-\xi\mu &= \beta \mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}\left[1-\delta+(1-\mu')F_k(z',k',n')\right]\right), && (10)\\
(1-\xi\mu)q &= \beta \mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}\left[1-D'\lambda\right]\right). && (11)
\end{aligned}
$$

Equation (9) is the optimality condition for labor, where the marginal productivity of labor, $F_n(z,k,n)$ is equalized to the marginal cost. The marginal cost is the wage rate w augmented by a wedge that depends on the tightness of the enforcement constraint, $\frac{1}{1-\mu}$. If the enforcement constraint is not binding, i.e., $\mu = 0$, then the optimality condition is reduced to the standard condition where the marginal product of labor equals the wage rate. If the constraint is tighter (a higher μ), it increases the effective cost of labor and reduces labor demand. A channel through which default risks are transmitted to the real sector of the economy is through the reduction in the demand for labor.

When the government defaults, it reduces banks' assets. This constrains banks' ability to obtain funds from households and to lend to firms. In the model, as well as in the data, when banks suffer a negative shock to their assets, banks have to contract their lending to firms. With less working capital loans, firms in turn have to cut down on production.

To understand the impact of default further, I rewrite the enforcement constraint. Using the budget constraint (7) to substitute in $k' + qb'$, the enforcement constraint (8) can be

[13] We can think of this entity as the conglomerate that engages in both financial and nonfinancial businesses, or simply the representative private sector.

[14] The detailed derivation is provided in Appendix C.

rewritten as

$$\frac{\xi}{1-\xi}\left[(1-\delta)k+(1-D\lambda)b-wn-d\right]\geq F(z,k,n).$$

At the beginning of the period, k and b are given. Suppose the enforcement constraint is binding and the government defaults. Then the firm has to reduce dividend d and/or cut employment n.

In principle, the firm could relax the enforcement constraint by issuing negative dividends, so as to avoid the cost of reducing employment. But the firm only partially implement this strategy, because there is an indirect cost to change dividend.[15] From the budget constraint, due to the reduced supply of government bonds after a default, a reduction in dividend implies that the firm has to increase its investment by about the same amount. As will be shown later, the government tends to default when the TFP is low. Increasing investment when the TFP is low further depresses the return to capital. Increasing investment also reduces the stochastic discount factor in equilibrium. If the required increase in investment to make the enforcement constraint lax is too much, the firm is not willing to do so. Therefore the firm uses a combination of reducing dividend and reducing employment in response to a default.

3 The Government's Problem

I now proceed to define the government's problem. In each period, the government chooses tax, debt and default policies to maximize the welfare of households, while subject to two conditions. First, government budget constraint has to be satisfied; and second, the resulting allocations have to constitute a competitive equilibrium as defined earlier. I consider government policies that are time consistent, using the concept of a Markov perfect equilibrium, as in Klein, Krusell and Rios-Rull (2008).

Intuitively, one can think of the economy as having a different government each period. Each successive government chooses only current policy, taking future governments' policies as given. In other words, today's government cannot directly choose future policies. Instead, it needs to form an expectation about future governments' policy rules when making decisions. The equilibrium is characterized as the fixed point in the policies rules, whereby the current government does not have incentives to deviate from future governments' policy rules, thus making these rules time consistent.

Due to the representative-agent framework, from now on, I will denote the aggregate capital and debt by lower-case letters as long as they do not cause confusion. At the beginning

[15] Adding direct dividend adjustment cost would reinforce the mechanism here.

of every period, the government decides if it is going to default,

$$\begin{aligned}
V(z,k,b) &= \max\left\{V^r(z,k,b), V^d(z,k,b)\right\} \\
D=0 \quad \text{if} & \quad V^r(z,k,b) \geq V^d(z,k,b) \\
D=1 \quad \text{if} & \quad V^r(z,k,b) < V^d(z,k,b)
\end{aligned}$$

where V^r is the value of repaying and V^d is the value of defaulting. If the government repays, it solves the following problem:

$$\begin{aligned}
V^r(z,k,b) &= \max_{c,n,d,k',b',\tau,w,q,\mu} U(c,n) + \beta\mathbb{E}\left[V(z',k',b')\right] \\
&\text{subject to} \\
c &= (1-\tau)wn + d \\
d + k' + qb' &= (1-\delta)k + zk^{\alpha}n^{1-\alpha} - wn + b \\
gzk^{\alpha}n^{1-\alpha} + b &= qb' + \tau wn \\
\frac{U_n}{U_c} &= -(1-\tau)w \\
(1-\alpha)zk^{\alpha}n^{-\alpha} &= \frac{w}{1-\mu} \\
(1-\xi\mu)q &= \beta\mathbb{E}\left(\frac{U_c(s')}{U_c}\left[1 - D(s')\lambda\right]\right) \\
1-\xi\mu &= \beta\mathbb{E}\left(\frac{U_c(s')}{U_c}\left[1-\delta+(1-\mu(s'))\alpha z'k'^{\alpha-1}n(s')^{1-\alpha}\right]\right) \\
\xi(k'+qb') &\geq zk^{\alpha}n^{1-\alpha}, \mu \geq 0, \text{ and } \mu[\xi(k'+qb') - zk^{\alpha}n^{1-\alpha}] = 0
\end{aligned}$$

where $s' = (z', k', b')$, and the government takes as given the future government's policy rules and future competitive equilibrium allocation rules. Here, the competitive equilibrium conditions are subsumed into the value function as constraints to the government's problem.[16] It is as if the government chooses all allocations in the economy. But the presence of these constraints guarantees that these allocations are the competitive equilibrium allocations.

[16]This setup is similar to the primal approach in the Ramsey problem.

Similarly, if the government defaults, it solves the following problem:

$$\begin{aligned}
V^d(z,k,b) &= \max_{c,n,d,k',\tau,w,q,\mu} U(c,n) + \beta \mathbb{E}\left[V(z',k',(1-\lambda)b)\right] \\
&\text{subject to} \\
c &= (1-\tau)wn + d \\
d + k' + q(1-\lambda)b &= (1-\delta)k + zk^{\alpha}n^{1-\alpha} - wn + (1-\lambda)b \\
gzk^{\alpha}n^{1-\alpha} + (1-\lambda)b &= q(1-\lambda)b + \tau wn \\
\frac{U_n}{U_c} &= -(1-\tau)w \\
(1-\alpha)zk^{\alpha}n^{-\alpha} &= \frac{w}{1-\mu} \\
(1-\xi\mu)q &= \beta \mathbb{E}\left(\frac{U_c(s')}{U_c}\left[1 - D(s')\lambda\right]\right) \\
1-\xi\mu &= \beta \mathbb{E}\left(\frac{U_c(s')}{U_c}\left[1-\delta+(1-\mu(s'))\alpha z' k'^{\alpha-1} n(s')^{1-\alpha}\right]\right) \\
\xi(k' + q(1-\lambda)b) &\geq zk^{\alpha}n^{1-\alpha}, \mu \geq 0, \text{ and } \mu[\xi(k'+q(1-\lambda)b) - zk^{\alpha}n^{1-\alpha}] = 0
\end{aligned}$$

where $s' = (z', k', (1-\lambda)b)$. For both value functions V^r and V^d, the first constraint is the household's budget constraint, the second is the bank's budget constraint, the third is the government budget constraint, the fourth is the household's labor supply equation, the fifth is the firm's demand for labor, the sixth and seventh are the bank's intertemporal Euler equations for bonds and capital respectively, the last constraints are the enforcement constraint and the complementary slackness conditions (the Kuhn-Tucker conditions). Now we are ready to define the Markov perfect equilibrium.

Definition 2: Let $s = \{z, k, b\}$. A Markov perfect equilibrium is defined as a set of functions for (1) value functions $\{V(s), V^r(s), V^d(s)\}$; (2) policy functions $\{D(s), \tau(s), b'(s)\}$; (3) allocation rules $\{c(s), n(s), d(s), k'(s)\}$; (4) pricing functions $\{w(s), q(s), \mu(s)\}$ such that:

1. given pricing functions, allocation rules and future government policy functions, current government policy functions $\{D(s), \tau(s), b'(s)\}$ solve the government's maximization problems $\{V(s), V^r(s), V^d(s)\}$;

2. given pricing functions and government policy functions, $\{c(s), n(s), d(s), k'(s)\}$ are the competitive equilibrium allocation rules;

3. policy functions obtained by solving the government problems coincide with future government policy functions that the government problems take as given;

4. markets clear.

3.1 Tradeoffs in Government Policies

In this subsection, I discuss the tradeoffs associated with the government's policies. I first start with the equilibrium condition for labor, by combining the labor supply equation (5) and the labor demand equation (9) into

$$-\frac{U_n(c,n)}{U_c(c,n)} = (1-\tau)(1-\mu)F_n(z,k,n).$$

$-\frac{U_n}{U_c}$ is the marginal rate of substitution and F_n is the marginal product of labor. Following the tradition in the business cycle accounting literature[17], I refer to $(1-\tau)(1-\mu)$ as the labor wedge, which in this case is a combination of distortions due to taxes (τ) and financial frictions (μ is the tightness of the enforcement constraint).

First, we consider increasing the amount of debt. Issuing more debt (higher b') reduces current tax rate (lower τ) and relaxes the enforcement constraint (lower μ). However, higher b' implies a higher tax rate next period. Thus, the government faces this intertemporal tradeoff of decreasing current distortion at the expense of increasing future distortion. Furthermore, there is a limit to which current distortion can be reduced. As the government has no commitment and can default on its debt, higher debt implies higher default risk and lower bond price q. Thus, when the debt level is high enough, increasing debt may even reduce debt revenue and increase current tax.[18] Similarly, reducing the debt level has the opposite effects.

Second, we consider defaulting on debt. As the government does not need to pay back a fraction of its debt, it has the benefit of reducing current tax rate (lower τ). But it undermines the firm's ability to raise working capital by tightening the enforcement constraint. Reducing the tax rate increases labor supply of households, while tightening the enforcement constraint decreases labor demand of firms. Default has this intratemporal tradeoff. Furthermore, default has an additional benefit of starting off the economy with a lower level of debt in the next period. As the model features a closed economy, it does not include the conventional tradeoff where default represents a net transfer of resources from foreigners.

Next, we consider the government policies' impact on investment and capital. Due to arbitrage, the expected returns on capital and bonds have to be equalized. On the negative side, increasing debt crowds out capital, as more resources are diverted away from investment in capital to the purchase of government bonds. On the positive side, bond provides an additional means of financing working capital loans, therefore effectively loosens

[17] See Chari, Kehoe and McGrattan (2007).

[18] It is as if the economy is on the right side of the Laffer curve.

the enforcement constraint.[19]

To formalize the above idea, we look at the Euler equation for capital (equation (10), reproduced below for convenience),

$$\frac{1-\xi\mu}{\beta} = \mathbb{E}\left(\frac{U_{c'}}{U_c}\left[1-\delta+(1-\mu')F_{k'}\right]\right).$$

The left-hand side is the expected return to capital, which is lower than the rate of time preference, $\frac{1}{\beta}$, when the enforcement constraint is binding. Hence, increasing bond holdings increases the amount of collateral and raises the expected return to capital. On the right-hand side of the equation, μ' acts like a tax rate on capital income, although there is no explicit capital tax in the model. As higher amount of bonds means the future enforcement constraint will be less binding (smaller μ'), increasing debt effectively reduces future capital income tax. However, as default makes the enforcement constraint more binding, it decreases the return to capital and increases effective capital tax. Therefore, in the model, sovereign default shifts the tax burden from labor to capital, so default can also be viewed as a tradeoff between labor wedge and investment wedge.

4 Calibration

In this section I describe the data and the calibration procedure, and comment on the solution method. I calibrate the model to the Argentine economy for the period of 1980-2010. Argentina is a typical emerging economy studied in the literature because it has defaulted several times in modern history.

I collect annual data on national accounts, employments and public debt for Argentina from 1980-2010. Data on GDP, households' consumption and investment are from the World Development Indicators by the World Bank. Data on government expenditure is from the Latin American and Caribbean Macro Database by the Inter-American Development Bank (IADB). Data on employments and worked hours is from the Conference Board Total Economy Database. IADB Database and Panizza (2008) both provide public debt data and a breakdown into domestic and external public debt. I merge their data and find that Argentina has an average domestic debt to GDP ratio of 27%.[20]

The model is calibrated to an annual frequency. All the parameter values are listed in Table 1. Household's discount factor β is set to 0.95, a common value in the real business

[19]This is similar to the idea in Aiyagari and McGrattan (1998), where government bonds relax the borrowing constraints of households.

[20]The average total public debt to GDP ratio in Argentina is about 60%.

cycle literature with annual frequency. Household's utility function takes the form of

$$U(c,n) = \ln(c) - \chi \frac{n^{1+\nu}}{1+\nu}$$

which is consistent with the preferences used in the growth and real business cycle literature. To have worked hours equal to 0.33,[21] the parameter for disutility of labor χ is calibrated to be 4.18. The curvature of labor supply ν is chosen to match the Frisch elasticity of labor supply, $1/\nu$. The estimates for this elasticity vary considerably in the literature. I choose ν to be 0.5, implying a Frisch elasticity of 2, within the range of estimates in the macro literature. I conduct sensitivity analysis on ν later.

Table 1: Parameters for Argentina

Calibrated Parameters		Value	Target Statistics
Household's discount factor	β	0.95	Standard value
Disutility of labor	χ	4.18	Steady state hours = 0.33
Curvature of labor supply	ν	0.5	Frisch elasticity = 2
Capital share in output	α	0.3	Standard value
Capital depreciation rate	δ	0.1	Investment/GDP = 20%
Government spending/GDP	g	0.23	Govt spending/GDP = 23%
Partial default	λ	0.55	Haircut = 55%
Enforcement parameter	ξ	0.440	Mean debt/GDP = 27%
Autocorr. of productivity shock	ρ_z	0.813	Autocorr. of TFP = 0.813
Std. dev. of productivity shock	σ_z	0.0459	Std. dev. of TFP = 0.0459

In the Cobb-Douglas production function, the capital share α is set to 0.3, a standard capital share in GDP.[22] The capital depreciation rate δ is chosen to be 0.1 to match an average investment/GDP ratio of 20% over the period 1980-2010. The government spending to GDP ratio g is set to 0.23, reflecting an average ratio of General Government Expenditures to GDP of 23% in Argentina. For the partial default parameter λ, the empirical counterpart is the haircut associated with defaults. Benjamin and Wright (2013) find an average haircut of 51%, and Sturzenegger and Zettelmeyer (2008) estimate a range of 13% to 73%. I set λ equal to 0.55 in the baseline calibration and investigate the sensitivity of results under different values of λ.

[21] This number is obtained by normalizing the average hours worked per year per worker by $(24-8)\times 365$.

[22] The capital share is 0.3 in the Italian data. I take the same number for Argentina so as to make comparisons easier.

The enforcement parameter ξ studied in this paper is fairly abstract and has no direct empirical counterpart. For this reason, I follow the tradition in the financial frictions literature and use the model's implications to relate the tightness of the enforcement constraint to a set of observable variables. Recall that the enforcement constraint is $\xi(k' + qb') \geq F(z, k, n) = y$. Suppose the constraint is binding at the deterministic steady state, I can rewrite it as

$$\xi \left(\frac{k}{y} + q\frac{b}{y} \right) = 1$$

I first guess the parameter ξ and solve the model. It generates a value for $\frac{b}{y}$ at the steady state. I then match this value of $\frac{b}{y}$ to the debt/GDP ratio in the data and verify the constraint is indeed binding at the steady state. The average of domestic public debt to GDP ratio in Argentina is 0.27. The required value is $\xi = 0.440$.[23]

To calibrate the autocorrelation ρ_z and the standard deviation σ_z of the productivity shock, I follow the standard Solow residuals approach. I use annual data on GDP, investment, and labor from 1960 to 2010 in Argentina to construct a series for Solow residuals.[24] I assume the shock to z follows an AR(1) process,

$$\ln(z_{t+1}) = \rho_z \ln(z_t) + \epsilon_{z,t}$$

where $\epsilon_{z,t}$ is i.i.d. with standard deviation σ_z. I estimate the equation and get $\rho_z = 0.813$ and $\sigma_z = 0.0459$. I then use Tauchen (1986) quadrature method to construct a Markov approximation to this process.

As analytical solutions are not available for the model, I use numerical methods to analyze the model's behavior. The model is solved using a global solution that relies on projection methods. In particular, I use policy function iteration to solve for competitive equilibrium and value function iteration to solve for the government problem. As the enforcement constraint is not always binding, I also need to check for occasionally binding constraints.[25] Appendix D provides a detailed description of the algorithm.

[23] The assumption that the amount of working capital loan is equal to the value of output is not crucial. The analysis carries through as long as the working capital loan is positively related to the scale of production. Suppose, for example, the working capital loan required is a fraction θ of the output. In other words, the enforcement constraint is $\xi(k + qb) \geq \theta y$. Calibration then requires that $\frac{\xi}{\theta}\left(\frac{k}{y} + q\frac{b}{y}\right) = 1$. The baseline model is simply setting $\widehat{\xi} = \frac{\xi}{\theta}$.

[24] I use the longer time series here to construct the capital stock and TFP process.

[25] Christiano and Fisher (2000) provide an algorithm for solving models with occasionally binding constraints.

5 Quantitative Analysis

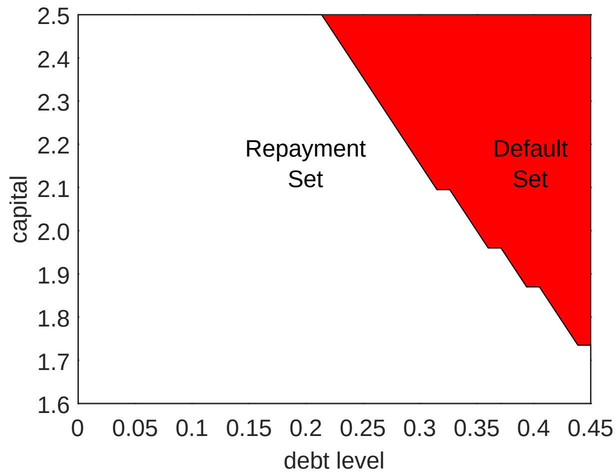
In this section I focus on the quantitative results of the model by applying it to the Argentine economy. Specifically, I first examine the government policy rules, then analyze the implications for default, present the dynamics around the default episodes, and last conduct some sensitivity analysis.

5.1 Policy Functions

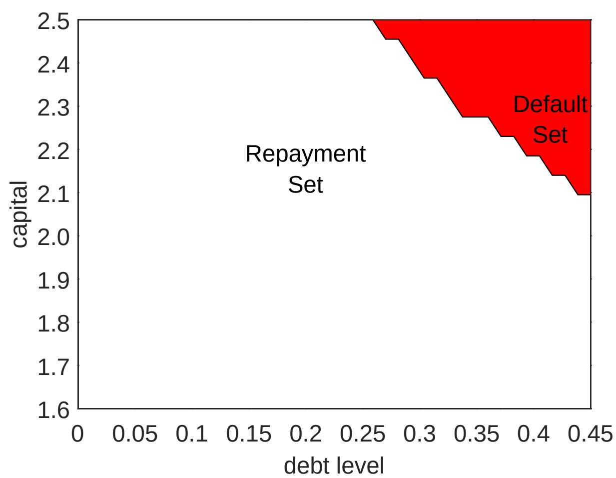
To highlight the role of different elements in the model, I analyze the government's optimal policies in this subsection. I begin with the sovereign's default decisions. Figure 3 shows the default and repayment sets of the economy for different combinations of capital stock and debt across different realizations of the productivity shock. Figure 3a depicts a low TFP shock, while Figure 3b and Figure 3c depict the mean and a high shock respectively. Across all figures, the x-axis is the debt level and the y-axis is the capital stock level. Both the debt level and the capital stock are normalized as ratios to output. The shaded region represents all combinations of debt and capital for which the economy is better off defaulting (the default set). The complement region represents all combinations for which repaying the debt is better (the repayment set).

Looking across the subfigures from (a) to (c), we observe that the default set shrinks as the TFP z increases. This result is in line with the empirical evidence that countries default more often in bad times. When the TFP is low, the cost of repaying the debt is higher, because the government has to charge a higher tax rate to repay the debt. As taxes are distortionary, increasing tax rate during downturn is more harmful to the economy relative to when the state of the economy is good. At the same time, the enforcement constraint is less binding when z is low as the amount of working capital required is smaller. This translates into a smaller default cost for lower values of z. Altogether, higher cost of repaying and lower cost of defaulting induce the government to default more often when the economy is hit with negative shocks.

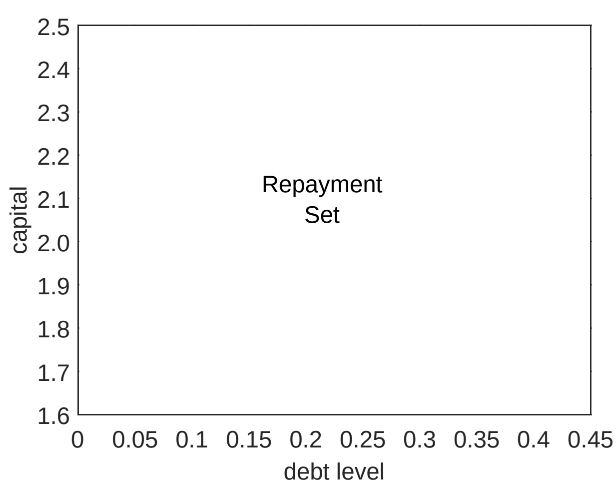
In order to produce the negative correlation between default risk and output, models with endowment economies (for example, Arellano (2008)) generally have to assume an ad-hoc asymmetric default cost. That is, default cost is low or even zero when output is low, and high when output is high. The reason is that when the output is high, the economy has less need for borrowing to smooth consumption. Thus there are greater incentives to default in economic expansions. To correct this anomaly, these models therefore have to assume an asymmetric default cost. In my model, the enforcement constraint is less binding in recessions because the amount of working capital needed is smaller. It thus endogenously



(a) Low productivity shock



(b) Mean productivity shock



(c) High productivity shock

Figure 3: Default and Repayment Sets

creates a default cost that is higher in expansions and lower in recessions.

Figure 3 shows a positive correlation between the default risk and the capital stock conditional on a given level of TFP and debt. In the model, physical capital and government bonds are substitutes in the sense that they provide liquidity by allowing banks to increase lending to firms. For a given level of bond holdings, a higher level of capital implies there is a larger amount of banks' assets. This relaxes the enforcement constraint and tames the severity of the contraction following a default. Thus, ceteris paribus, a higher capital stock makes default more likely.

The above result may seem somewhat surprising, as one may think that economies with more capital stock are wealthier and hence are able to sustain more debt.[26] However, if we compare developed and developing countries (rich and poor countries), they differ much more in their levels of TFP than in the levels of capital stock. More importantly, if we focus only on emerging economies, output and investment are typically growing above trend until a year before a default. It means that prior to default, these countries often have capital stocks that are above their respective trends. My model gives a clear explanation for this phenomenon, because the default cost is smaller if the economy has a larger capital stock.

The model also predicts a negative correlation between the default risk and the debt level conditional on a given level of TFP and capital, as depicted in Figure 3. This is a standard result because higher debt requires the government to charge higher taxes to repay the debt, which gives the government greater incentives to default.

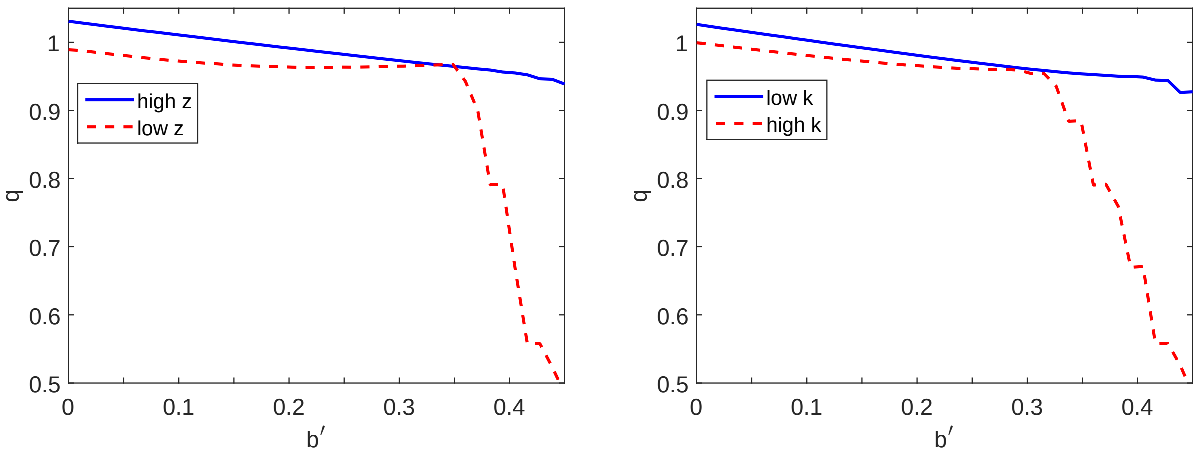
Figure 4 depicts the bond price q as a function of the next period debt b'. The left panel shows it for different values of productivity shock z while the right panel shows it for different values of capital stock k. Recall that bonds are priced according to

$$q = \frac{\beta}{1-\xi\mu}\mathbb{E}\left[\frac{U_{c'}}{U_c}\left(1-D'\lambda\right)\right].$$

There are two major departures from the bond pricing schedules in the sovereign default literature. First, the model features risk averse pricing while the literature typically assumes risk neutral pricing. As Lizarazo (2013) has shown, risk averse pricing is more relevant empirically and incorporating it in the model explains bond prices better than models with risk neutral investors. Second, there is a liquidity premium to hold bonds, represented by $\frac{1}{1-\xi\mu}$ in the above equation. The liquidity premium arises due to the fact that bonds help to relax the enforcement constraint. Bond prices are thus higher when the enforcement constraint is more binding ($\mu > 0$). In such states, government bonds are more valuable

[26] Gordon and Guerron-Quintana (2013) show that default is more likely to occur in economies with lower capital stock, because these economies have less income to repay their debts. However, in their model, default costs are exogenous and debt does not affect the productive capacity of the economy.

because they allow firms to obtain more funding and help to facilitate production.



(a) across z

(b) across k

Figure 4: Bond Pricing Functions

In the left panel of Figure 4, ceteris paribus, bond price q is higher for higher productivity z. This is because higher z implies higher consumption c and therefore lower marginal utility of consumption today relative to tomorrow (lower U_c relative to $U_{c'}$). High consumption today means households want to save more to smooth consumption. This increases the stochastic discount factor, decreases the interest rate, and increases the price of bonds. For high values of b', default risk tomorrow increases and therefore reduces the bond price today. As default risks are much higher for lower values of z and shocks are persistent, q falls at a greater rate when z decreases.

In the right panel of Figure 4, bond price is negatively correlated with capital stock. This is because capital and government bonds are substitutes in terms of satisfying the enforcement constraint. Mechanically, the enforcement constraint is more binding for lower capital stock. This increases the Lagrange multiplier μ (or the liquidity premium of bonds) and thus increases bond price q. Again, when b' is high, the default risk effect dominates, and q decreases. It decreases more sharply for higher k due to the positive correlation between capital and default probability.

5.2 Impact of Defaults

To assess the impact of defaults in the model, I compare moments from the data with moments from the model's simulations. With shocks drawn from the stationary distribution of z, I conduct 1000 simulations, each with 500 periods and truncate the first 100 observations.

Table 2 shows the statistics related to defaults. According to Reinhart and Rogoff (2011), domestic defaults are about 1/4 as frequent as external defaults.[27] As the probability of (total) defaults has been estimated to be about 3%,[28] the probability of domestic defaults is about 0.75%. In the model, the default rate is 0.78%, very close to the data.

Table 2: Default Episodes

	Data	Model
Default probability	0.75%	0.78%
Mean output drop	11%	11.6%
Mean investment drop	36%	33.0%
Correlation btw default and GDP	−0.11	−0.127
Fraction of defaults with large recessions	32%	32.1%
Fraction of defaults with GDP below trend	62%	91.4%

In Table 2, the average output drop during default is 11.6% in the model, a close match to those observed in the data, which is 11%. On average, investment decreases by about 33% in the model, while it is 36% in the data. The model is able to match these decreases in output and investment very well. These decreases are the endogenous default costs in the model. In both the data and the model, the fall in investment is more than three times that of the drop in output. This again highlights the importance of incorporating investment and capital in the model, which the previous default literature has not done.

Table 2 also reports the relationship between output and default based on the historical cross-country data listed by Tomz and Wright (2013) and Benjamin and Wright (2013). The correlation between defaults and output in the model (−0.127) is a good match to the correlation in the data (−0.11). The negative signs indicate that defaults occur in "bad times". This also holds true for "unusually large" recessions, defined as when GDP is two standard deviations below trend. In the data, the fraction of defaults that occur in large recessions is about 32%, while it is 32.1% in the model. Tomz and Wright (2013) also emphasize that although defaults tend to happen in "bad times", it is not always the case. They report the fraction of defaults that occur with GDP below trend is 62%, implying that in 38% of the times, defaults happen with GDP above trend. Although the model is able to generate some defaults when outputs are above trend, the fraction of defaults that occur with GDP below trend is 91% in the model, higher than that in the data.

[27] Based on Reinhart and Rogoff (2011), since 1800, there has been 68 domestic defaults and 250 external defaults.

[28] Researchers typically use the fact that Argentina defaulted three times in the past one hundred years, translating into a 3% annual default rate.

The model is able to sustain a debt to GDP ratio up to 35%, with reasonable default rate and discount factor ($\beta = 0.95$ in the model). This debt ratio is much higher than the ratios typically obtained in the sovereign default literature. For instance, with an endowment economy, Arellano (2008) reports a mean debt to output ratio of only 6%. While Aguiar and Gopinath (2006) obtain a ratio of 27%, they have a very low discount factor of 0.8 and generate an annual default rate of only 0.08%.[29] With a production economy, Mendoza and Yue (2012) report a debt to GDP ratio of 23%, but have a low quarterly discount factor of $\beta = 0.88$. The reason for sustaining a much higher debt level in this paper is due to the positive role of the debt in production in the model. In most other models, debt acts as the (only) means to smooth consumption but does not otherwise affect output. In this model, debt, in addition to capital, helps banks to transfer resources from households to firms. By allowing firms to raise more working capital, debt facilitates output production. Furthermore, government bonds also provide additional saving instruments in the economy. Otherwise, all the savings have to be in the form of physical capital, which would suppress the returns to capital. Defaulting on debt disrupts these mechanisms and hurts the economy, thus the model gives rise to a much higher debt ratio.

Table 3 reports the business cycle moments from the Argentine data with those produced by the model. Overall, the model matches output very well. Faced with the same TFP processes, the standard deviation of output (5.40%) in the model is only slightly less than that in the data (5.66%). Different from most of the sovereign default literature, I do not calibrate the model to match the output, but nevertheless, it produces a very close match.

Table 3: Simulation Results for Argentina

	Data	Model
σ_y	5.66%	5.40%
σ_c/σ_y	1.14	0.41
σ_i/σ_y	2.95	3.59
σ_n/σ_y	0.31	0.53
σ_{r^s}	2.51%	1.21%
$corr(y, c)$	0.89	0.51
$corr(y, i)$	0.87	0.96
$corr(y, n)$	0.36	0.60
$corr(y, r^s)$	-0.62	-0.64

[29] One exception is Chatterjee and Eyigungor (2012), who study long-term debt and produce debt/GDP ratio over 50%.

A stylized fact in emerging economy business cycle is that consumption variability exceeds output variability ($\sigma_c/\sigma_y = 1.14 > 1$ in the data). However, this ratio has a low value of 0.41 in the model. This is a feature of real business cycle models where they tend to generate low consumption volatility relative to output volatility (for example, see Backus, Kehoe and Kydland (1992)).[30] This is because when capital accumulation is allowed, investment becomes an important means to smooth consumption. In the data, investment is much more volatile than output (2.95) and there is a tight correlation between investment and output (0.87). In the model, investment to output volatility ratio is 3.59 and correlation is 0.96. When the TFP shock is high, investment increases by a large amount. This has two implications. First, it does not increase consumption as much and thus reduces consumption volatility. Second, it increases capital stock so that the economy has more buffer to withstand negative shocks in the future, therefore increases consumption in the periods when TFPs are low. Hence, incorporating capital in the model greatly reduces consumption volatility.

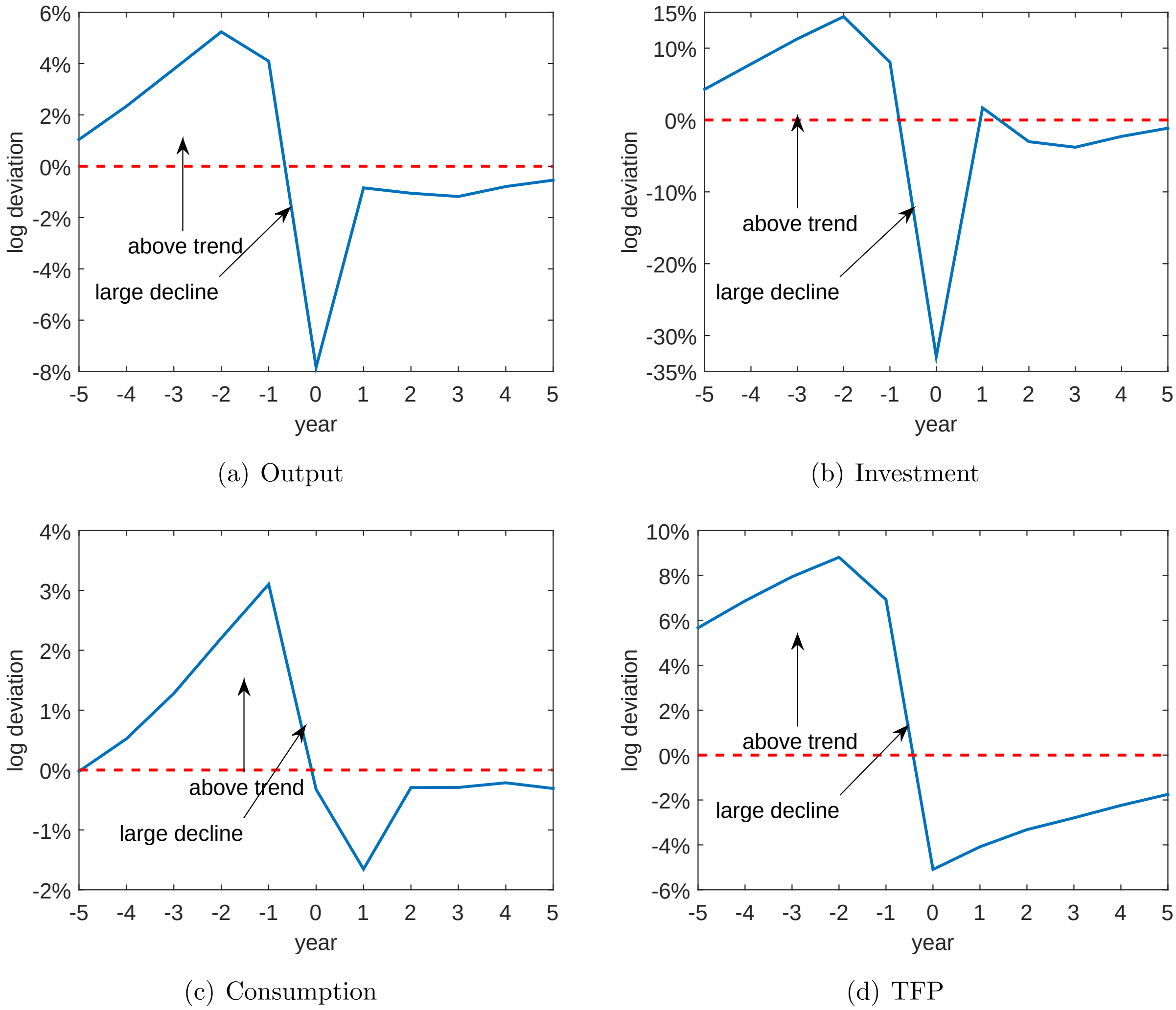
Interest rate spread is defined as the difference between the interest rate paid by the government and a risk-free interest rate. Recall that the government bond price is $q = \frac{\beta}{1-\xi\mu}\mathbb{E}\left[\frac{U'_c}{U_c}(1-D'\lambda)\right]$. I therefore define a risk-free bond price as $q^f = \frac{\beta}{1-\xi\mu}\mathbb{E}\left[\frac{U'_c}{U_c}\right]$, and the interest rate spread as $r^s = \frac{1}{q} - \frac{1}{q^f}$. The volatility of spreads is lower in the model than in the data. Models of endogenous default often underestimate this moment, and the variability produced here is similar to the one produced in Mendoza and Yue (2012). The model yields a correlation between spreads and output that is very close to the data. Interest rate spreads are highly counter-cyclical. As shown earlier, when productivities are low, defaults are more likely to occur. Hence, investors demand a higher return to hold government bonds, and spreads are consequently higher in bad times.

5.3 Dynamics around Default Events

In order to analyze the behavior of the model's dynamics around sovereign default episodes, I compute the average deviations of output, investment, consumption and productivity from their respective trends across default occurrences in the simulated time series data. First, I take log of the simulated data and hp-filter them to get log deviations; second, I identify the simulation periods where default happens; third, I construct a time series of five years before and five years after each default; fourth, I average across default episodes to construct

[30] It is a puzzle in international economics that emerging economies have high consumption volatility relative to output volatility and models in general do not generate this property. Some literature, for example Arellano (2008), produce this feature because they do not allow for capital and investment, which takes away an important means of consumption smoothing. In the data, investment is usually three to four times more volatile than output.

a series of mean log deviations from trends. Figure 5 shows the results, with the default periods normalized to date 0.



(a) Output (b) Investment

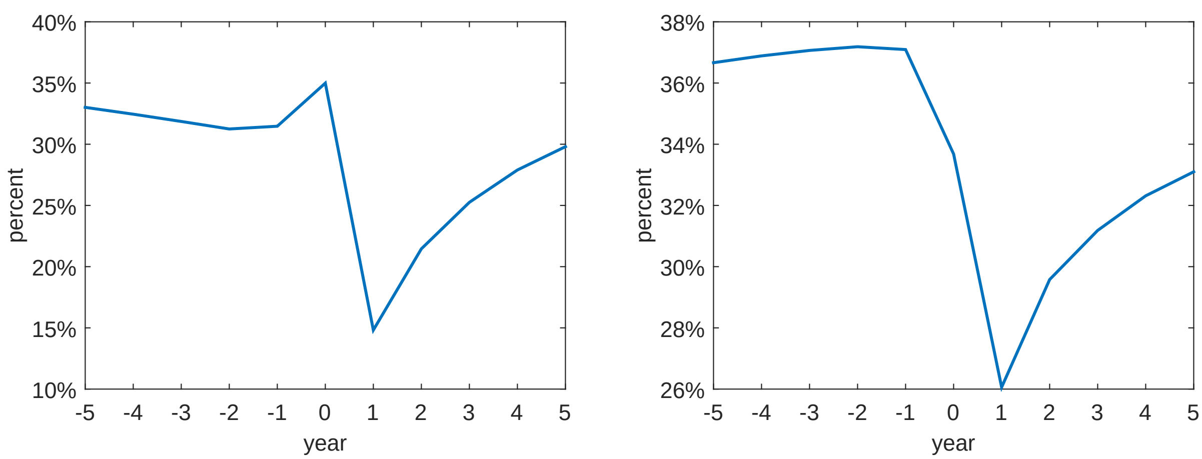
(c) Consumption (d) TFP

Figure 5: Default Dynamics

The model is able to endogenously account for the declines in output, investment and consumption around default episodes, triggered by the exogenous decline in TFP. Qualitatively, it is able to deliver the v-shaped behavior of all three variables around defaults, which is consistent with data. The mean declines in output and investment are 11.6% and 33% respectively, very close to the empirical counterparts, which are 11% and 36% respectively. Furthermore, in line with data, the model produces an investment deviation from trend of magnitude about three times that of the output. When the economy encounters negative shocks, investment decreases in order to smooth consumption. Despite the significant decline in savings, consumption decreases as well, but the percentage decrease is much smaller than those in output and investment.

Furthermore, the model successfully replicates the path leading up to a default. As illustrated in Figure 5, all variables grow above trend until one year before default. This is the general pattern of defaults observed in emerging economies. In the years preceding a default, countries are typically growing well, building up capital stock through increased investment. In the model, as the economy receives a sequence of positive shocks (Figure 5d), it increases investment to reap the benefits of increased productivity. If the economy then encounters a large negative shock, the cost of default is smaller, because the high level of capital stock built up in earlier periods implies that banks have more assets to withstand the negative shock.

To illustrate the government's strategic behavior more clearly, let us take a look at Figure 6, which depicts the dynamics of debt and tax rate around default. The debt to output ratio reaches the peak in the period of default. This is also the period of low productivity and output. If the government were to repay the debt in this period, it has to charge a higher tax rate. Due to the concavity of utility function, costs of tax distortions are convex. Therefore, tax distortions are high in periods of low productivity/output, and reducing distortions has greater benefits when taxes are high. This gives the government stronger incentives to default. Coupled with a lower cost of default as explained in the previous paragraph, the government thus finds it optimal to default at the moment of low productivity and high capital stock. In Figure 6b, we see that the tax rate is reduced in the period of default. It is further reduced in the period immediately following default, as the economy starts off with a lower debt level and is able to borrow again by issuing new debt.



(a) Debt/GDP

(b) Tax

Figure 6: Debt and Tax

Some limitations of the model should be acknowledged. First, consumption volatility relative to output volatility is much smaller in the model than in the data. As explained earlier, this is a feature of the real business cycle model to generate low consumption volatility. The reason is because with capital accumulation, investment increases or decreases to smooth consumption. Second, recovery after default is faster in the model than in the data. The reason is that the economy starts off with a much lower level of debt after default. With access to credit market, the government has incentives to immediately increase debt. One benefit is that it reduces current tax rate and thus increases households' labor supply. Furthermore, raising debt relaxes the enforcement constraint, thereby allowing firms to take more working capital loan and increase labor demand. The combined forces accelerate the economy's recovery rate.

5.4 Sensitivity Analysis

In this section, I conduct sensitivity analysis to evaluate the quantitative performance of the model with respect to changes in the main parameters. The results are summarized in Table 4. The first two rows reproduce the statistics from the data and the baseline model. The rest of the table shows the statistics for each alternative scenario, which include changes to the partial default rate λ, the Frisch elasticity of labor supply $\frac{1}{\nu}$, and the enforcement parameter ξ.

Table 4: Sensitivity Analysis

	Default probability	Output drop	Investment drop	$\frac{\sigma(c)}{\sigma(y)}$	$\frac{\sigma(i)}{\sigma(y)}$	$\sigma(r^s)$	$corr(y,c)$	$corr(y,i)$	$corr(y,r^s)$
Data	0.75%	11%	36%	1.14	2.95	2.51	0.89	0.87	−0.62
Baseline	0.78%	11.6%	33.0%	0.41	3.59	1.21	0.51	0.96	−0.64
Partial default rate (baseline: $\lambda = 0.55$)									
$\lambda = 0.5$	1.15%	10.7%	33.1%	0.42	3.60	1.54	0.49	0.96	−0.56
$\lambda = 0.6$	0.57%	12.1%	33.6%	0.41	3.56	1.22	0.54	0.96	−0.68
$\lambda = 0.7$	0.28%	12.9%	34.3%	0.41	3.49	1.17	0.61	0.96	−0.74
Frisch elasticity (baseline: $\frac{1}{\nu} = 2$)									
$\frac{1}{\nu} = 3$	0.45%	12.6%	35.8%	0.41	3.62	0.87	0.52	0.96	−0.66
$\frac{1}{\nu} = 2.5$	0.62%	11.9%	33.7%	0.41	3.59	1.03	0.52	0.96	−0.67
$\frac{1}{\nu} = 1.5$	0.95%	11.1%	31.8%	0.44	3.56	1.30	0.50	0.96	−0.68
Enforcement constraint (baseline: $\xi = 0.44$)									
$\xi = 0.43$	0.63%	11.6%	31.6%	0.42	3.57	1.34	0.52	0.96	−0.75
$\xi = 0.435$	0.66%	11.9%	33.1%	0.41	3.59	1.31	0.52	0.96	−0.69
$\xi = 0.445$	0.85%	11.4%	33.4%	0.43	3.58	1.02	0.50	0.96	−0.63
$\xi = 0.45$	0.94%	10.8%	32.1%	0.42	3.59	1.01	0.51	0.96	−0.52

I first consider how the model responds to changes in the partial default rate λ. When the government can default on its debt at a higher haircut (higher λ), the costs of a default are increased. This is because a higher haircut implies a greater loss of banks' assets, which makes the enforcement constraint tighter and further reduces the amount of working capital loans that firms can raise. Consequently, output losses are greater and the government therefore defaults less frequently. In Table 4, as the haircut on debt is increased from 50% to 70%, we see that the default frequency is reduced from 1.15% to 0.28%, while the falls in output and investment are increased from 10.7% and 33.1% to 12.9% and 34.3% respectively.

Next, I examine the changes in the Frisch elasticity of labor supply, $\frac{1}{\nu}$. As labor supply becomes more elastic (or as ν decreases), it implies that when a default occurs and wage declines due to higher costs of financing, there is a larger decrease in equilibrium labor. Hence, with higher labor elasticity, there is a greater output drop. At the same time, the default probability becomes smaller, as larger output fall means higher costs of default. Therefore in Table 4, when labor elasticity decreases from 3 to 1.5 (or equivalently, when ν increases from 0.333 to 0.667), the default probability increases from 0.45% to 0.95%, and the drop in output decreases from 12.6% to 11.1%.

Last, I consider the role of ξ, the enforcement constraint parameter. This parameter ξ governs the tightness of the enforcement constraint. Recall that the enforcement constraint is $\xi(k' + qb') \geq \ell$. Conditional on banks' assets, the higher the value of ξ, the higher the amount of working capital loans that firms can obtain. Hence the enforcement constraint is less binding, and government bonds are less useful in facilitating production. In this case, when the government defaults, the negative impact on output is smaller. Consequently, with a smaller default cost, the government is tempted to default more often. Therefore, when ξ is increased from 0.43 to 0.45, we see that the default probability increases from 0.63% to 0.94%, and the drop in output decreases from 11.6% to 10.8%.

5.5 Discussions

In this section, I briefly consider three extensions of the baseline model, and discuss the implications.

First, consider extending the model to an open economy. In the data, government bonds are usually held by both domestic and foreign investors, and the government defaults on both groups. If international investors were allowed to hold government bonds and they do not have interactions with domestic banks or firms, defaulting on them alone would not affect the output. But if the government cannot discriminate when it defaults or if international investors are linked to domestic banks or firms, the model can still generate output loss endogenously due to the balance sheet effect.

Second, suppose households are also allowed to hold government bonds. In this case, banks would still invest in bonds, as long as returns on bonds and physical capital are equalized. Again, if the government cannot discriminate, the balance sheet effect highlighted in the paper continues to hold. In the data, financial sectors typically hold a large share of government bonds, especially during crises. For example, Calvo and Mishkin (2003) argue that banks in Argentina increased holdings of government bonds during the 2001 Argentine crisis. This is a form of financial repression studied in Chari, Dovis and Kehoe (2016).

Third, consider equity injection during debt crises. The government or households can recapitalize banks by transferring resources to them when the government defaults. However, this alone will not undo the effects of a default. As explained in earlier sections, there is a shortage of collateral after a default, and banks are unwilling to increase investment by too much because of the low return to capital. Hence, the enforcement constraint continues to bind and output decreases. This is similar to the result in Bocola (2014) that the provision of liquidity to banks during the European debt crisis had limited effects because banks were reluctant to lend to firms during crisis.

6 Comparison of Italy and Argentina

In this section, I first apply the model to the Italian economy. Even though Italy has not (yet) defaulted, I attempt to answer questions on what would happen to its economy if Italy defaults. I then compare and contrast the results with that of the Argentine economy. The calibration strategy is the same, and the calibrated parameters for Italy are in Appendix E.

One purpose of the exercise is to shed light on the default risk in a developed economy. This aspect has been traditionally ignored in the sovereign default literature, but the European debt crisis shows that even developed countries are not immune from default risk. For this reason, I pick Italy because it is an important developed country that is exposed to default risk and it has a large domestic debt to GDP ratio around 60%. Another purpose is, by comparing an emerging and a developed country, to show that the model is consistent with both types of economies.

Table 5: Simulated Defaults in Italy

	Model
Default probability	0.028%
Mean output drop	6.0%
Mean investment drop	8.8%

Table 5 shows the simulation results for Italy. The simulated default probability in Italy is only 0.028% per year, almost negligible.[31] This is in fact consistent with the experience of Italy, which has never defaulted in modern history, although it is exposed to some risk in the European debt crisis. The simulated default rate for Argentina is 0.78%, about 30 times of that of Italy.

One major difference between Argentina and Italy is the TFP process. The TFP volatility in Argentina is 4.59%, while it is only 1.96% in Italy.[32] As the Argentine economy is much more likely to experience very negative shocks, it increases the probability of default, because the government has stronger incentives to default when the economy is in the bad state. In turn, with higher default probability, Argentina sustains a much lower debt ratio of 27% compared to Italy's 59%. In this model, different TFP processes in the two countries, coupled with the default costs generated endogenously through the presence of the enforcement constraint, are able to jointly account for higher (lower) default frequency and lower (higher) debt ratio in Argentina (Italy).[33]

In the European debt crisis, some of the major concerns for the policymakers and economists are the spread of default risk and (potential) disruptions to the real economy if involved countries default. I use the model to conduct a counterfactual analysis on what would happen to the Italian economy if it defaults. In Table 5, it shows that Italy would suffer an output drop of 6.0% and an investment drop of 8.8% if it defaults. Around 6% decrease in real GDP for a developed country is substantial. This may help to explain why Italy has not (yet) defaulted and apprehensions in Europe are not without reason.

6.1 Counterfactual Analysis on Argentina

To further understand the differences in default and debt sustainability between Italy and Argentina, I perform counterfactual analysis on the Argentine economy using parameters from the Italian calibration. Two major differences between Italy and Argentina, in terms of parameter values, are the enforcement constraint and the productivity process. I change these two sets of parameters, one at a time, from the Argentine values to the Italian values, while keeping all the other Argentine parameters fixed. Table 6 summarizes the results.

Starting from the Argentine model economy, I change the enforcement constraint pa-

[31] A default rate of 0.028% per year translates to less than three times in every 10,000 years.

[32] TFP volatilities in Argentina and Italy are in Table 1 and Table 7 respectively. Emerging economies in general encounter much more volatile exogenous shocks than developed economies, regardless of the types of shocks. For example, see Aguiar and Gopinath (2007) and Neumeyer and Perri (2005).

[33] Another major difference between Argentina and Italy is the level of financial frictions in the economy, represented by the enforcement constraint and the parameter ξ. I examine this difference in the next subsection.

Table 6: Counterfactual Analysis on Argentina

	Default probability	Output drop	Investment drop	$\frac{\sigma(c)}{\sigma(y)}$	$\frac{\sigma(i)}{\sigma(y)}$	$\sigma(r^s)$	$corr(y,c)$	$corr(y,i)$	$corr(y,r^s)$
Data	0.75%	11%	36%	1.14	2.95	2.51	0.89	0.87	−0.62
Baseline	0.78%	11.6%	33.0%	0.41	3.59	1.21	0.51	0.96	−0.64
Argentina has Italy's financial friction: $\xi = 0.39$									
	0.27%	11.8%	26.2%	0.43	3.53	2.05	0.54	0.96	−0.79
Argentina has Italy's productivity process: $\rho_z = 0.925, \sigma_z = 0.0196$									
	0.24%	4.6%	5.8%	0.45	3.11	0.57	0.75	0.96	−0.50

rameter ξ from the Argentine value of 0.44 to the Italian value of 0.39. The purpose is to ask that, if Argentina has the level of financial frictions as that of Italy, what would happen to Argentine defaults? As shown in Table 6, the default frequency decreases to 0.27%, while output loss experiences a slight increase to 11.8%. Intuitively, when ξ decreases, the enforcement constraint becomes more binding. It increases the costs of default and makes default less likely. In the experiment, although the increase in default costs in terms of output loss is insignificant, the decrease in default frequency is substantial.

Again starting from the baseline Argentine model, I change the productivity process parameters to the Italian values of $\rho_z = 0.925$ and $\sigma_z = 0.0196$. This exercise shows that, if Argentina has the Italian TFP process, it can lower the default probability to 0.24% and reduce the output loss to only 4.6%. As explained earlier, the government is more prone to default when the economy encounters a large negative shock after a series of positive shocks. If Argentina is able to reduce its TFP volatility, it would be less likely to be in such a situation where the government finds it better off to default on its debt. Furthermore, a lower TFP volatility leads to a smaller output variance. Thus when Argentina defaults in this scenario, the fall in output is reduced, because the corresponding decrease in TFP, which triggered default, is smaller.

In summary, these counterfactual exercises on Argentina suggest that if Argentina's banking sector is more closely connected to its real sector, it would increase Argentina's default costs, thereby allowing Argentina to sustain more debt and default less. But perhaps more importantly, if Argentina is able to improve on its TFP by making it less volatile, not only would it allow Argentina to default less frequently, but also to experience a much smaller output decline upon default.

7 Conclusion

This paper develops a model of sovereign default in a production economy with financial frictions, within which output and default risk are jointly determined. The framework allows it to simultaneously examine the behaviors of output and investment, and their interactions with sovereign debt and default. In the model, firms need to finance working capital, which they use to hire workers and make production plans. Firms borrow from banks, which in turn borrow from households. The amount of loans that can be transferred from households to firms is limited by banks' assets, which consist of physical capital and government bonds. However, the government chooses tax and debt policies without commitment. If the government defaults, it reduces banks' assets, thereby reducing loans to firms and forcing firms to reduce labor and decrease production. But paying back outstanding debt requires the government to levy distorting taxes. The government weighs the costs of reducing output and the benefits of lowering tax distortion when it makes the optimal default decisions.

This paper captures some salient features of sovereign defaults. First, a high level of debt can be sustained in the economy. This is because debt provides liquidity and enhances production in the model. Defaulting on debt undermines the productive capacity of the economy and is therefore very costly. Second, key aggregate variables are above trend until default. This is because the government is more likely to default if the economy experiences a sequence of good productivity shocks followed by a large negative shock. After a sequence of good shocks, investment increases and the economy ends up with a higher level of capital stock. Cost of default becomes smaller as there are more capital to withstand negative shocks. What triggers default then is a drop in productivity, because tax burden is higher when productivity/output is low. Third, as the result of the mechanisms described, the model endogenously generates declines in output and investment around default episodes.

There are a number of interesting extensions to the paper. A first extension is to consider an open economy. Adding this dimension helps to account for both domestic and external public debt and generate implications for international capital flows. Second, with endogenous default costs, we can use the framework to evaluate government intervention policies during debt crisis. The effects of an intervention policy depend on its impact on output and welfare, and endogenous default costs provide a basis to compare policies. Third, endogenizing haircut decisions can help to explain heterogeneous default events and understand what conditions induce the governments to take higher haircuts on debt.

Appendix

A Data Description

I describe the variables and data sources for the cross-country data used in the paper.

Default Events: I consider 23 default events. The list is from Mendoza and Yue (2012): Argentina (1982, 2002), Chile (1983), Croatia (1992), Dominican Republic (1993), Ecuador (1999), Indonesia (1998), Mexico (1982), Moldova (2002), Nigeria (1983, 1986), Pakistan (1998), Peru (1983), Philippines (1983), Russia (1998), South Africa (1985, 1993), Thailand (1998), Ukraine (1998), Uruguay (1990, 2003), Venezuela (1995, 1998).

Real GDP: Gross domestic product at constant local currency unit. Data are annual, seasonally adjusted, 1960-2013. The source is World Bank's World Development Indicators.

Real Consumption: Household final consumption expenditure at constant local currency unit. Data are annual, seasonally adjusted, 1960-2013. The source is World Bank's World Development Indicators.

Real Investment: Gross capital formation at constant local currency unit. Data are annual, seasonally adjusted, 1960-2013. The source is World Bank's World Development Indicators.

Employment: Number of persons employed. Data are annual, seasonally adjusted, 1960-2013. The source is the Conference Board Total Economy Database.

Worked Hours: Average number of hours worked per year per worker. Data are annual, seasonally adjusted, 1960-2013. The source is the Conference Board Total Economy Database.

B Derivation of the Enforcement Constraint

Figure 7 illustrates the renegotiation game between a bank and a lender. The bank borrows an intraperiod loan ℓ from the lender. If the bank decides to repay, the lender gets back the loan and the bank continues its operation. At this moment, the payoff to the bank is $Em'W'$, where m' is the stochastic discount factor and W' is the next-period value of the bank. The payoff to the lender is the loan ℓ.

If the bank decides not to repay, the lender acquires the right to liquidate the bank's assets. Suppose the lender can acquire a fraction ξ of the bank's assets $k' + qb'$. However, the bank can renegotiate with the lender by offering the lender an amount of x. If the lender accepts this offer, the lender will not liquidate the bank and has a payoff of x. The bank keeps the intraperiod loan and keeps its operation, therefore its payoff is $Em'W' + \ell - x$ at

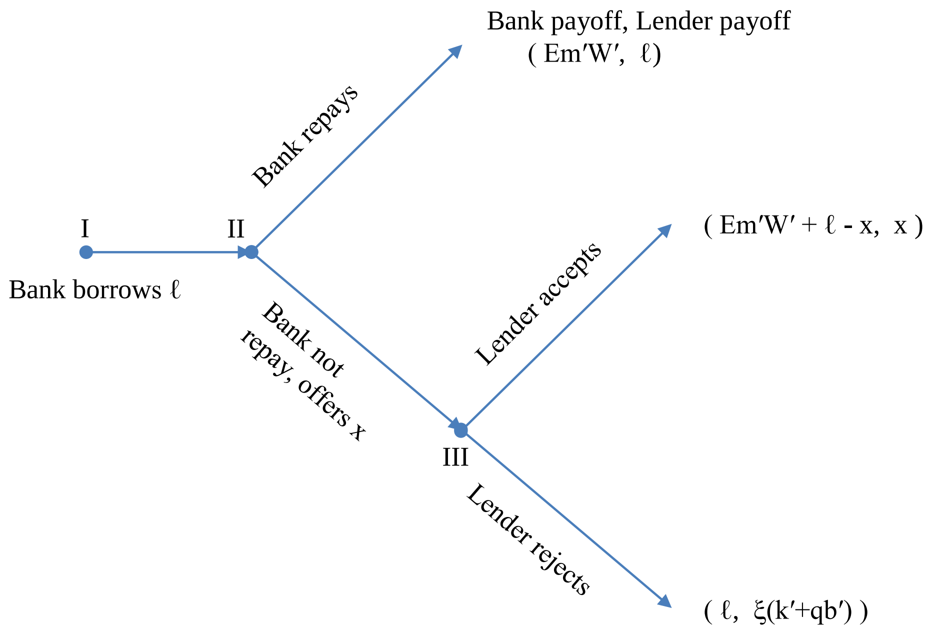


Figure 7: Renegotiation Game

this moment. But if the lender rejects the offer, the bank is liquidated and has only ℓ in hand, while the lender gets the liquidation value of $\xi(k' + qb')$.

Since it is in the interest of the bank to continue its operation, the bank has to make an offer that leaves the lender indifferent between liquidating and keeping the bank in operation. This requires the bank to offer $x \geq \xi(k' + qb')$ in order for the lender to accept.

Hence, if the intraperiod loan is more than the liquidation value of the bank, i.e. $\ell > \xi(k' + qb')$, the bank will choose not to repay and instead offer $\xi(k' + qb')$ to the lender. Anticipating this behavior, the lender is only willing to lend

$$\ell \leq \xi(k' + qb'),$$

which gives the enforcement constraint in the model.

C First-Order Conditions

For convenience, we reproduce the firm's problem here.

$$
\begin{aligned}
W(k,b;S) &= \max_{d,n,k',b'} d + \beta\mathbb{E}\left[\frac{U_c(c',n')}{U_c(c,n)}W(k',b';S')\right], \\
&\text{subject to} \\
d + k' + qb' &= (1-\delta)k + F(z,k,n) - wn + (1-D\lambda)b, \\
\xi(k'+qb') &\geq F(z,k,n).
\end{aligned}
$$

Let θ and μ be the Lagrange multipliers on the budget constraint and the enforcement constraint respectively. The first-order conditions with respect to d, n, k' and b' are

$$
\begin{aligned}
1-\theta &= 0, \\
\theta(F_n - w) - \mu F_n &= 0, \\
\theta - \xi\mu &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}W_{k'}\right), \\
(\theta - \xi\mu)q &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}W_{b'}\right).
\end{aligned}
$$

The envelope conditions are

$$
\begin{aligned}
W_k &= \theta(1-\delta+F_k) - \mu F_k, \\
W_b &= \theta(1-D\lambda).
\end{aligned}
$$

Combining the first-order conditions and the envelope conditions, we get

$$
\begin{aligned}
F_n(z,k,n) &= \frac{w}{1-\mu}, \\
1-\xi\mu &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}\left[1-\delta+(1-\mu')F_k(z',k',n')\right]\right), \\
(1-\xi\mu)q &= \beta\mathbb{E}\left(\frac{U_c(c',n')}{U_c(c,n)}\left[1-D'\lambda\right]\right).
\end{aligned}
$$

D Algorithm

The numerical solution of the model is based on a global solution that uses projection method. It consists of value function iteration for the government problem and policy function iteration for the competitive equilibrium. As the enforcement constraint is not always binding, I also need to check for occasionally binding constraints.

The model solves for the optimal time consistent fiscal policy,[34] so the solution searches for the fixed point where neither the government nor the agents have incentives to deviate from their respective policies. The idea is that given the current and future governments' policies, agents in the economy adjust their expectations and make their decisions. In turn, based on agents' decisions and the future government's policies, the current government chooses its policies to maximize agents' welfare. The government's policies are time consistent if the current and future governments' policy functions coincide. For ease of elaboration, I denote the conditional expectations by E^k and E^b where

$$\begin{aligned} E^b(z', k', b') &= \mathbb{E}\left(U_c(c', n')\left[1 - D'\lambda\right]\right) \\ E^k(z', k', b') &= \mathbb{E}\left(U_c(c', n')\left[1 - \delta + (1 - \mu')\alpha z' k'^{\alpha-1} n'^{1-\alpha}\right]\right) \end{aligned}$$

and rewrite the government's value function when it does not default,

$$\begin{aligned} V^r(z, k, b) &= \max_{c,n,d,k',b',\tau,w,q,\mu} U(c, n) + \beta \mathbb{E}\left[V(z', k', b')\right] \\ &\text{subject to} \\ c &= (1 - \tau)wn + d \\ d + k' + qb' &= (1 - \delta)k + zk^\alpha n^{1-\alpha} - wn + b \\ gzk^\alpha n^{1-\alpha} + b &= qb' + \tau wn \\ \frac{U_n}{U_c} &= -(1 - \tau)w \\ (1 - \alpha)zk^\alpha n^{-\alpha} &= \frac{w}{1 - \mu} \\ (1 - \xi\mu)qU_c &= \beta E^b(z', k', b') \\ (1 - \xi\mu)U_c &= \beta E^k(z', k', b') \\ \xi(k' + qb') &\geq zk^\alpha n^{1-\alpha}, \mu \geq 0, \text{ and } \mu[\xi(k' + qb') - zk^\alpha n^{1-\alpha}] = 0 \end{aligned}$$

The government's value function of defaulting is similar.

The algorithm contains two loops: the inner loop iterates on the government's value functions while the outer loop iterates on the agents' expectations. The details of the algorithm consist of the following steps:

1. Create grids for productivity shocks, capital stock and bond holdings, $[\underline{z}, \overline{z}] \times [\underline{k}, \overline{k}] \times [\underline{b}, \overline{b}]$.

[34] Klein, Krusell and Rios-Rull (2008) solves for a time consistent tax policy in a deterministic environment with one asset.

2. Make initial guesses for V^0, $E^{k,0}$, and $E^{b,0}$.

3. Suppose we are at the i-th iteration.

4. At each grid point (z, k, b) and for each choice of b', I first assume the enforcement constraint is binding and solve a system of eight equations (the eight constraints in the value function) with eight unknowns $\{c, n, d, k', \tau, w, q, \mu\}$ using a nonlinear equation solver based on a variant of the Newton's method.

5. If the multiplier μ is negative, I set it to zero, drop the enforcement constraint and solve the system of seven equations and seven unknowns.

6. The set of variables $\{c^{*,r}(b'), n^{*,r}(b'), d^{*,r}(b'), k^{'*,r}(b'), \tau^{*,r}(b'), w^{*,r}(b'), q^{*,r}(b'), \mu^{*,r}(b')\}$ are the economy's competitive equilibrium conditions if the government does not default and chooses b'.

7. In a similar fashion, I solve for the economy's competitive equilibrium conditions if the government defaults.

8. Given these solutions, I calculate the welfare $V^d(z, k, b)$ and $V^r(z, k, b) = \max_{b'} \widehat{V}^r(z, k, b; b')$, and choose the optimal $b^{'*} \in \text{ argmax } \widehat{V}^r(z, k, b; b')$.

9. Use the results in step 8 to choose optimal default decision: $D^* = 1$ if $V^d > V^r$ and $D^* = 0$ otherwise.

10. Use the results in step 8 to update $V(z, k, b) = \max\left\{V^r(z, k, b), V^d(z, k, b)\right\}$.

11. Repeat step 8 to step 10 until the value function V converges. Denote it by V^i and the associated policies by $b^{'i}$ and D^i.

12. Obtain competitive equilibrium conditions. For example, $c^i = c^{*,d}$ if $D^i = 1$ and $c^i = c^{*,r}(b^{'i})$ if $D^i = 0$.

13. Use the values in step 12 to update agents' conditional expectations. Denote them by $E^{k,i}$ and $E^{b,i}$.

14. Repeat step 4 to step 13 until the expectations E^k and E^b converge.

E Parameters for Italy

I collect annual data on national accounts, worked hours and public debt for Italy from 1980-2010. Data on GDP, households' consumption, investment, government expenditure are from the OECD National Accounts. Data on worked hours and labor income share are from the EU KLEMS database. Data on public debt is from the OECD database on central government debt. It has a breakdown of total public debt into domestic and external debt. The calibration strategy is the same as that for Argentina, and the calibrated parameter values are in Table 7.

Table 7: Parameters for Italy

Calibrated Parameters		Value	Target Statistics
Household's discount factor	β	0.95	Standard value
Disutility of labor	χ	4.27	Steady state hours = 0.32
Curvature of labor supply	ν	0.5	Frisch elasticity = 2
Capital share in output	α	0.3	Labor income share = 0.7
Capital depreciation rate	δ	0.1	Investment/GDP = 20%
Government spending/GDP	g	0.21	Govt spending/GDP = 21%
Partial default	λ	0.55	Haircut = 55%
Enforcement parameter	ξ	0.390	Mean debt/GDP = 59%
Autocorr. of productivity shock	ρ_z	0.925	Autocorr. of TFP = 0.925
Std. dev. of productivity shock	σ_z	0.0196	Std. dev. of TFP = 0.0196

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