Employment, Wages and Optimal Monetary Policy

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Abstract

We investigate optimal monetary policy when the policymaker considers both a model with search and matching frictions in the labor market and a model with sticky nominal wages to be plausible approximations of the true data-generating process. Although the models imply similar impulse response functions for common variables under an estimated interest rate rule, these responses differ importantly when policy is set optimally in each model. Price inflation is tempered in the search and matching model at the expense of wage inflation, with the reverse being the case in the sticky wage model. Employing the concept of optimal targeting rule, we show that a policy optimal in one model is far from optimal in the other model. In other words, the optimal targeting rules are not robust. When monetary policy takes into account model uncertainty, policy shifts its focus away from price inflation as long as the policymaker assigns a moderate probability to the sticky wage model. Our findings provide all the more reason for central banks to put stronger emphasis on wage inflation dynamics contrary to their public deliberations.

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1 Introduction

Macroeconomic models intended to capture the cyclical properties of employment and wages often feature either nominal wage rigidities or search and matching frictions in the labor market. We study optimal monetary policy when policymakers view both approaches as providing a reasonable fit to the data.

The empirical New Keynesian (NK) literature has largely relied on the idea of nominal wage rigidities to construct models with labor market dynamics that are in line with stylized facts of the business cycle while setting the individual labor supply elasticity at realistic values. Fine illustrations of this approach are Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005). Models with search and matching frictions in the labor market as in Diamond (1982), Mortensen (1982), and Pissarides (1985), by contrast, assume nominal wages to be flexible. A worker's (real) wage is not determined in an anonymous market, but it is the result of a bargaining game between a worker and a firm. Although common in macroeconomics, search and matching frictions are rarely embedded in monetary models of the business cycle. Notable exceptions are Krause and Lubik (2007) and Ravenna and Walsh (2008).[1]

In recent work, Christiano, Eichenbaum, and Trabandt (2013) find that a NK model with search and matching frictions fits the macro data just as well as a NK model with sticky nominal wages when monetary policy follows an estimated interest rate rule.[2] We consider this finding to be potentially troubling news as the two models have conflicting normative implications for monetary policy. It is known that the optimal monetary policy places high weight on stabilizing wage inflation at the expense of price inflation when nominal wages are sticky, but it places high weight on price inflation when nominal wages are flexible and search and matching frictions prevail in the labor market.[3] This tension sets the stage for our study of robust monetary policy with multiple reference models. We show that unless the NK model with sticky wages can be ruled out with near certainty, monetary policy should be less concerned with price inflation and instead

[1] Gertler, Sala, and Trigari (2008), and Gertler and Trigari (2009) merge the two strands of literature by replacing the assumption of period-by-period wage bargaining in favour of staggered multi-period wage contracts.

[2] This interpretation of the findings in Christiano, Eichenbaum, and Trabandt (2013) is not necessarily shared by the authors of that study. They stress in the abstract that under on their adopted statistical decision criterion "Our model outperforms a variant of the standard New Keynesian Calvo sticky wage model." Yet, in a probabilistic sense, the evidence presented by the authors would not lead one to believe that one of the models is correct with certainty.

[3] For optimal monetary policy analysis in a model with sticky nominal wages similar to ours see Erceg, Henderson, and Levin (2000); for the case of search and matching frictions Ravenna and Walsh (2011) is the closest reference.

stabilize the relevant wage dynamics.

Our version of the NK model with search and matching frictions in the labor market builds on Faia (2009) and Ravenna and Walsh (2011). As in their works, wholesale firms post vacancies and workers search for jobs. When a firm and a worker are matched, they Nash bargain over the terms of employment and production of wholesale output occurs. Retail firms differentiate wholesale output and set prices for varieties using staggered contracts as in Calvo (1983). However, similar to Sveen and Weinke (2009), we assume that the labor supply of an employed worker is elastic and individual hours worked are determined during the bargaining process along with the real wage. The reasons for this departure are twofold. First, this assumption facilitates comparability with standard sticky wage models which feature an elastic labor supply at the individual level; for certain parameter choices the two models nest each other. Second, our approach models explicitly the opportunity costs of employment, a key element in improving the model's fit of the empirical patterns of unemployment and vacancies.[4] Our NK model with sticky nominal wages builds on Erceg, Henderson, and Levin (2000) with minor modifications. Labor is differentiated and households set wages for each labor variety in a staggered fashion. Sticky prices are modelled as in the search and matching model.

To ensure that the two models generate dynamics in line with selected empirical evidence, we employ an impulse response function matching estimator. In particular, we estimate the coefficients of the interest rate rule adopted by the monetary policymaker. For each model we compute the optimal monetary policy under commitment from the timeless perspective in response to technology and price markup shocks — a task not undertaken in Christiano, Eichenbaum, and Trabandt (2013).[5] Following the vast literature on optimal monetary policy, the policymaker's preferences coincide with those of the representative household in the model. In the model with search and matching frictions price inflation is kept under tight control while nominal wages display large movements under the optimal policy.[6] Although the search and matching process is not efficient in

[4] Shimer (2005) argues that search and matching models cannot generate labor market movements that are in line with the empirical evidence for plausible parameter choices. Numerous authors have tackled this criticism: Hall (2005) and Shimer (2005) propose real wage rigidities; Hagedorn and Manovskii (2008) argue in favor of high opportunity costs of employment; Hall and Milgrom (2008) suggest departures from Nash bargaining over wages. Yashiv (2007) provides a comprehensive summary of the debate and a broader assessment of the search and matching framework. Our approach keys off the ideas in Hagedorn and Manovskii (2008), but we model the opportunity costs of employment explicitly and allow for them to vary over the business cycle.

[5] See Woodford (1999) for a discussion of "optimality from the timeless perspective."

[6] Apparently, the labor market variables behave rather differently under the estimated interest rate rule than socially desirable. This contrast is not restricted to real and nominal wages, but includes unemployment and vacancy postings.

our setting, monetary policy cannot correct the underlying distortions in the labor market. Thus, the optimal monetary policy focuses on addressing the dynamic distortions associated with sticky prices in the product market. Low inflation reduces the differences in relative prices across varieties and the associated inefficient shifts in relative demand. The degree of inflation stabilization is only constrained by the possible trade-off between inflation and resource utilization (as measured by output gaps). By contrast, in the NK model with sticky nominal wages, the optimal policy needs to strike a balance between price and wage inflation. Similar to the product market, wage inflation distorts relative real wages and labor demand in the labor market; price inflation supports the adjustment of real wages under staggered nominal wages. The near complete stabilization of wages reflects the high welfare costs associated with even minor relative wage differences in empirical sticky wage models.

We then investigate how each model economy would fare if it is exposed to the optimal policy associated with the other model. To conduct this experiment, we employ the concept of optimal targeting rules discussed in depth in Giannoni and Woodford (2016). The optimal targeting rule specifies the variables — including the relative importance and the dynamic structure of each variable — in a single target criterion that seeks to implement the optimal monetary policy. In other words, the optimal targeting rule is a commitment to a certain relationship between the model variables. If the optimal targeting rule for the NK model with sticky nominal wages is implemented in the model with search and matching frictions, monetary policy restricts wage inflation drastically at the expense of more volatile price inflation; the dynamics of all endogenous variables in the search and matching model are far from the dynamics under the optimal policy. Similar results obtain when the optimal targeting rule of the search and matching model is imposed in the sticky wage model: price inflation in the sticky wage model becomes more stable than under the optimal policy and wage inflation features much larger movements.

Apparently, a targeting rule that is optimal in one model can be far from being optimal in the other model as indicated by welfare losses that are orders of magnitudes larger than the welfare costs of business cycles in Lucas (2003). The lack of robustness of the optimal targeting rules is not necessarily symmetric though. The optimal targeting rule derived form the search and matching model induces welfare losses in the sticky wage model that are ten times larger than in the opposite case. A policymaker who knows with absolute certainty which of the two models constitutes the true data-generating process would never choose the optimal targeting rule derived from the wrong model.

However, a policymaker may not have a unique approximating model in mind. In fact, our empirical approach is constructed in a way that does not allow to resolve model uncertainty via standard model selection exercises prior to the evaluation of monetary policy, but model uncertainty is a component of that evaluation along the lines argued in Brock, Durlauf, and West (2007). We analyze monetary policy when the policymaker acknowledges multiple models and pursues either a Bayesian or a minmax strategy to assess the optimality of a policy. In this part of the analysis, we restrict attention to simple interest rate rules to keep the policymaker's problem of finding the optimal parameterization of the rule manageable.[7]

Under a Bayesian strategy, the policymaker searches for an instrument rule that minimizes the expected loss for a given probability distribution over the relevant reference models. In some sense, the policymaker's model is a weighted average across the reference models. Unless the policymaker is very certain about the search and matching model being the correct data-generating process, the optimal interest rate rule resembles the optimal targeting rule *derived in the sticky nominal wage model*. In other words, the optimal targeting rule derived from the search and matching model is not robust to the (small) possibility that the sticky wages model is the true data-generating process. This lack of robustness is more pronounced if price markup shocks play a more prominent role in the two models. At the same time, the consequences of implementing policies that stabilize wage inflation in the search and matching model are benign.

When the policymaker adopts a minmax strategy, the optimal interest rate rule minimizes the maximum expected loss. This approach does not require the policymaker to assign specific probability weights to the models; for a model to be included in the list of reference models it suffices that the policymaker assigns non-zero probability to it being the true data-generating process. The optimal interest rate rule in this case mimics the optimal targeting rule derived in the sticky wage model.

Our approach to model uncertainty is closest to Levin and Williams (2003) and Levin, Wieland, and Williams (2003) which study robust monetary policy with competing reference models. Brock, Durlauf, and West (2007) outline basic principles for incorporating model uncertainty into policy evaluation exercises. Other related papers include Cogley and Sargent (2005) and Svensson and Williams (2005), but we abstract from the learning

[7] The simple interest rate rules are flexible enough to mimic closely the optimal targeting rules absent model uncertainty in both models.

dynamics featured in these contributions. In sync with our conclusions, these works reveal the principle that policymakers concerned with avoiding worst case scenarios should shy away from tailoring their policies towards a model with recommendations that are not robust to model misspecification and uncertainty even if the model is very likely.[8]

Our analysis differs from all these contributions along important dimensions. First, we restrict attention to microfounded models and exclude macro-econometric models from the set of reference models. Thus, we can derive an objective function of the policymaker that is consistent with the underlying reference models and reflects the policymaker's probability distribution over the models. The aforementioned contributions assume that the policymaker's preferences are independent of the models under consideration. Second, we parameterize the models to fit the same empirical evidence under empirical interest rate rules *before* deriving the optimal monetary policy. In Cogley and Sargent (2005) and Svensson and Williams (2005), model parameters are estimated conditional on the policymaker setting policy to maximize a given quadratic objective; no two models fit the data equally well over a given historical episode and the ranking of the models according to the quality of fit switches between episodes. In Levin and Williams (2003) and Levin, Wieland, and Williams (2003) the models are not parameterized using the same empirical evidence. Third, labor market aspects are at the core of our analysis and we stress the importance of smoothing wage dynamics at the expense of price inflation as a general principle of robust optimal monetary policy. These considerations are ruled out in the earlier contributions by the choice of models. Sensitivity analysis suggests that our results survive if the policymaker's preferences resemble those of earlier studies and are common across models as long as the policymaker has sufficient dislike for price inflation.

The remainder of the paper proceeds as follows. Section 2 presents the NK model with search and matching frictions and the NK model with sticky nominal wages. Section 3 discusses the details of our empirical strategy to parameterize the two models. Section 4 derives optimal targeting rules for each model and assesses the robustness of these rules across models. Optimal instrument rules under model uncertainty when policymakers follow Bayesian and minmax strategies are discussed in Section 5. Concluding remarks are offered in Section 6.

[8] Research on model uncertainty and policy evaluation has taken several directions. One direction is to assume a given baseline model and consider all models within a given distance as in Hansen and Sargent (2007), Tetlow and von zur Muehlen (2001), and Giannoni (2002). The second approach, taken in this paper, does not require that the models are close to each other. Another recent example of this approach is Taylor and Wieland (2012). In addition to model uncertainty, data uncertainty and parameter uncertainty are other areas of concern for policymakers.

2 Two competing models of the labor market

The two reference models that the policymaker considers in our framework of optimal monetary policy under model uncertainty build on the New Keynesian (NK) model with sticky nominal prices; the models differ with regard to the details of the labor market. The first model features search and matching frictions in the labor market as in Diamond (1982), Mortensen (1982), and Pissarides (1985). Each worker negotiates the terms of employment with the matched firm. While the real wage may adjust slowly to shocks, the nominal wage is fully flexible. By contrast, the second model introduces sticky nominal wages along the same lines as sticky nominal prices as in Erceg, Henderson, and Levin (2000). Unlike NK models with Walrasian labor markets, the two models we employ in the analysis are shown to fit well the impulse responses of labor market variables derived from structural vector auto-regressions (SVAR) for reasonable parameter choices. We provide brief model descriptions in the main text and refer to Appendix A for details.

2.1 NKM with search and matching frictions

Households are modelled as in Andolfatto (1996) and Merz (1995). At any point in time n_t agents of the household are employed (w) and $1 - n_t$ agents are unemployed (u). The household maximizes the weighted inter-temporal utility of agents

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[n_t U(c_t^w, h_t^w) + (1 - n_t) U(c_t^u, h_t^u) \right] \tag{1}$$

subject to the budget constraint

$$[n_t c_t^w + (1 - n_t) c_t^u] + \frac{B_{t+1}}{P_t} \leqslant [w_t h_t^w n_t + b^u (1 - n_t)] + \frac{\mathrm{Pr}_t}{P_t} + \frac{T_t}{P_t} + \frac{R_{t-1} B_t}{P_t}. \tag{2}$$

$\mathbb{E}_0$ is the expectations operator conditional on all the information available up to period 0. β is the time discount factor. Consumption is denoted by c_t^i, and hours worked by h_t^i, where $i = \{w, u\}$. The real wage is given by w_t and unemployment benefits (generated through home production) are measured by b^u as in Ravenna and Walsh (2011).[9] Bond holdings B_t, taxes and transfers T_t, and profits Pr_t are measured in nominal terms and

[9] We choose to model unemployment benefits through home production as this setup in principle allows for positive unemployment benefits and efficiency of matching as in Hosios (1990). While unimportant for this paper, this modelling choice plays a role in our companion paper Bodenstein and Zhao (2016).

are converted into real units through division by the price level P_t. R_t is the nominal interest rate on bonds. We denote by λ_t the Lagrange multiplier attached to the budget constraint when solving the household's problem.

To allow for as many similarities between the search and matching model and the sticky wage model as possible, individual preferences over consumption and hours worked are

$$U(c_t^i, h_t^i) = \frac{(c_t^i - hc_{t-1}^i)^{1-\sigma}}{1-\sigma} - \phi_0^i \frac{({h_t}^i)^{1+\phi}}{1+\phi} \tag{3}$$

with $\phi_0^u = 0$, i.e., unemployed agents do not experience disutility from searching for employment.

The labor market features search and matching frictions. Firms post vacancies v_t and u_t measures the share of agents searching for jobs. New matches m_t between firms and agents are formed according to the matching function

$$m_t = \chi u_t^\zeta v_t^{1-\zeta} \tag{4}$$

leading employment n_t to evolve according to

$$n_t = (1-\rho)\, n_{t-1} + m_t \tag{5}$$

where ρ is the exogenous rate at which existing matches break up. The number of job seekers in period t follows

$$u_t = 1 - n_{t-1} + \rho n_{t-1} = 1 - (1-\rho)\, n_{t-1}. \tag{6}$$

Wholesale firms employ labor as the only factor to produce the wholesale good y_t^w that is sold at the competitive market price P_t^w. To hire workers, wholesale firms have to first post a vacancy at the cost κ^v.[10] These firms maximize profits subject to the law of motion for employment and the production technology

$$\max_{\{n_t, y_t, v_t\}_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left(\frac{P_t^w}{P_t} y_t^w - \frac{W_t}{P_t} n_t h_t - \kappa^v v_t \right)$$
$$s.t. \quad n_t = (1-\rho)\, n_{t-1} + q_t v_t$$

[10] We abstract from fixed costs of starting the negotiation process suggested in Pissarides (2009) as our empirical procedure ruled out such costs.

$$y_t^w = a_t n_t h_t \tag{7}$$

where firms take the probability of filling an open vacancy $q_t = \frac{m_t}{v_t}$ as given. Total factor productivity a_t follows a standard AR(1) process

$$\log(a_t) = \rho_a \log(a_t) + \varepsilon_t^a \tag{8}$$

with normally distributed innovations $\varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$.

When an agent and a firm are matched, they engage in Nash bargaining over wages and hours worked. We assume Nash bargaining. The solution to the bargaining problem is obtained from

$$\max_{w_t, h_t} J_t^{1-\xi} H_t^{\xi} \tag{9}$$

where ξ stands for the bargaining power of the agent. The marginal value of employment to the firm J_t is given by the period profit of the additional worker, i.e., the excess of the marginal product over the real wage payment, plus the continuation value if the match survives into the next period

$$J_t = \left(\frac{P_t^w}{P_t} a_t - \frac{W_t}{P_t} \right) h_t + (1-\rho) E_t \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}. \tag{10}$$

The marginal value of employment to the household H_t satisfies

$$H_t = \left(\frac{W_t}{P_t} h_t - b^u - \frac{\phi_0}{1+\phi} \frac{h_t^{1+\phi}}{c_t^{-\sigma}} \right) + (1-\rho) E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - s_{t+1}) H_{t+1} \tag{11}$$

and consists of the increase in household income $\dfrac{W_t}{P_t} h_t - b^u$ by having an additional household member employed over the monetary equivalent to compensate the now employed member for the loss of leisure time $\dfrac{\phi_0}{1+\phi} \dfrac{h_t^{1+\phi}}{c_t^{-\sigma}}$ as well as the continuation value if the match survives into the next period.

Retail prices experience nominal rigidities. Retail firms produce differentiated goods using wholesale goods as the sole input. The optimization problem of retail firm i is split into two parts. The cost minimization problem is given by

$$\min_{y_t^w(i), Y_t(i)} P_t^w y_t^w(i)$$

$$s.t. \quad Y_t(i) = y_t^w(i). \tag{12}$$

This problem delivers an expression for the retailer's real marginal costs mc_t

$$mc_t = \frac{P_t^w}{P_t}. \tag{13}$$

Retailer i adjusts its price $P_t(i)$ each period with the fixed probability $1 - \xi^p$. For firms that do not re-optimize their price in a given period, prices will be updated as a weighted average of $\Pi_t = \frac{P_t}{P_{t-1}}$, the nominal price inflation in the previous period, and $\bar{\Pi}$, the steady state inflation rate

$$P_{t+1}(i) = \tilde{P}_t(i) \left({\Pi_t}^{\iota_p} \bar{\Pi}^{1-\iota_p} \right). \tag{14}$$

Retail firm i sets its price to maximize

$$\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi^p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left[\left((1+\bar{\tau}^p) \tilde{P}_t(i) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^p} \bar{\Pi}^{1-\iota^p} \right) - MC_{t+s} \right) \right] Y_{t+s}(i)$$

$$s.t. \quad Y_{t+s}(i) = \left(\frac{\tilde{P}_t(i) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^p} \bar{\Pi}^{1-\iota^p} \right)}{P_{t+s}} \right)^{-\dfrac{\lambda^p}{\lambda^p - 1}} Y_{t+s} \tag{15}$$

where $\bar{\tau}^p$ is a subsidy to the firm to offset distortions in the steady state due to monopolistic competition. We introduce a markup shock directly into the first order condition of the retailer. In choosing its price, the firm takes into account the demand curve for its differentiated good. This demand curve is derived from the problem of the producers of the final composite consumption good.

The differentiated goods are combined to form the composite good Y_t

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\lambda^p}} \mathrm{d}i \right]^{\lambda^p} \tag{16}$$

that is sold under perfect competition. The term $\frac{\lambda^p}{\lambda^p - 1}$ refers to the elasticity of substitution between the retail varieties.

2.2 NKM with sticky nominal wages

The model with sticky nominal wages differs from the search and matching model with regard to the labor market details. In this model, all household members are employed and nominal wages are set in staggered contracts following Calvo (1983). Each household j chooses consumption and asset holdings by maximizing the inter-temporal utility function

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{\left(c_t(j)-hc_{t-1}(j)\right)^{1-\sigma}}{1-\sigma}-\phi_0\frac{h_t(j)^{1+\phi}}{1+\phi}\right] \tag{17}$$

subject to the budget constraint

$$P_tc_t(j)+B_{t+1}(j)=\left(1+\bar{\tau}^w\right)W_t(j)h_t(j)+R_{t-1}B_t(j)+Pr_t(j)+T_t(j). \tag{18}$$

Labor bundlers package differentiated labor services supplied by each individual into an aggregate labor service offered at the aggregate nominal wage W_t. The labor bundling technology satisfies

$$h_t=\left[\int_0^1 h_t(j)^{\frac{1}{\lambda^w}}\mathrm{d}j\right]^{\lambda^w} \tag{19}$$

where the term $\frac{\lambda^w}{\lambda^w-1}$ measures the elasticity of substitution between differentiated labor services. The bundler's demand for variety j of labor services is given by

$$h_t(j)=\left[\frac{W_t(j)}{W_t}\right]^{-\frac{\lambda^w}{\lambda^w-1}}h_t. \tag{20}$$

Each household j supplies differentiated labor services $h_t(j)$ to the labor bundlers. The imperfect substitutability of differentiated labor services gives each individual household some market power. The household can readjust its wage with probability $1-\xi^w$ in each period. If the household cannot reoptimize its wages, wages will increase by a weighted average of past inflation and the steady state inflation rate according to

$$W_{t+1}(j)=\tilde{W}_t(j)\left(\Pi_t^{\iota^\omega}\bar{\Pi}^{(1-\iota^\omega)}\right). \tag{21}$$

A reoptimizing household chooses its wage as the solution to the following problem

$$
\begin{aligned}
& \max_{\tilde{W}_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi^w \beta)^s \left[\frac{(c_{t+s} - h c_{t-1+s})^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} h_{t+s}(j)^{1+\phi} \right] \\
s.t. \quad & P_{t+s} c_{t+s} + B_{t+s+1} = (1+\bar{\tau}^w) W_{t+s}(j) h_{t+s}(j) + R_{t+s-1} B_{t+s} + Pr_{t+s} + T_{t+s} \\
& h_{t+s}(j) = \left(\frac{W_{t+s}(j)}{W_{t+s}} \right)^{-\dfrac{\lambda^w}{\lambda^w - 1}} h_{t+s} \\
& W_{t+s}(j) = \tilde{W}_t(j) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^w} \bar{\Pi}^{1-\iota^w} \right) \qquad (22)
\end{aligned}
$$

Where $\bar{\tau}^w$ is the subsidy to households who supply differentiated labor varieties. If $\bar{\tau}^w = \lambda^w - 1$, labor supply distortions arising from monopolistic competition are eliminated.

Wholesale firms purchase aggregate labor services h_t from the labor bundler. Retail firms purchase the wholesale good, differentiate it, and set prices using staggered contracts, just as in the NK model with search and matching frictions.

2.3 Linearized models

We briefly turn to the log-linear approximation of the two models around their respective steady states. We restrict attention to our baseline specifications that abstract from price and wages indexation ($\iota^p = \iota^\omega = 0$) and consumption habits ($h = 0$) and we display the core equations only. The details for the search and matching model are provided in Appendix B; for the sticky wage model, we refer the reader to Erceg, Henderson, and Levin (2000).

Our derivations of the linear search and matching model resemble those in Ravenna and Walsh (2011) with two important exceptions: (i) the steady state is inefficient as we do not impose the conditions stated in Hosios (1990), (ii) the individual labor supply is elastic. Under the first assumption, the flexible price economy is not efficient. This feature of our model complicates finding the second order approximation to the preferences of the representative household which will be used in Section 4 to describe the preferences of the policymaker. The second assumption is of direct consequence for the linear presentation of the model's structural equations.

The linearized search and matching model can be reduced to three equations (excluding a description of monetary policy) and four endogenous variables (price inflation π_t, output $\hat{y}_t$, employment $\hat{n}_t$, and the nominal interest rate i_t all in deviations from the model's steady state values; "hatted" variables are in log-deviations). The exogenous shocks to technology ($\hat{a}_t$) and markups ($\hat{\theta}_{p,t}$) follow standard AR(1) processes. More specifically, the linear model is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa^p \left[\left(\phi + \sigma \frac{\varpi^{y_{ss}}}{1-\kappa^c} \right) \hat{y}_t - (1+\phi)\,\hat{a}_t - (\theta_1 \hat{n}_t + \theta_2 \hat{n}_{t-1}) \right] + \hat{\theta}_{p,t} \quad (23)$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1-\kappa^c}{\sigma \varpi^{y_{ss}}} \left(i_t - E_t \pi_{t+1} + (\theta_1 - \phi)\,(E_t \hat{n}_{t+1} - \hat{n}_t) + \theta_2\,(\hat{n}_t - \hat{n}_{t-1}) \right) \quad (24)$$

$$\gamma_1 E_t \hat{n}_{t+1} + \gamma_2 \hat{n}_t + \gamma_3 \hat{n}_{t-1} = \left(1 + \phi + \frac{\sigma \varpi^{y_{ss}}}{1-\kappa^c} \right) \hat{y}_t - (1+\phi)\,\hat{a}_t$$
$$- \frac{(1-\rho)\beta}{\mu} (1 - \xi q_{ss} \theta_{ss}) \left(\frac{\kappa^v}{q_{ss}} \right) (i_t - E_t \pi_{t+1}) \quad (25)$$

Equations (23) and (24) are the NK Phillips curve and the aggregate demand relationship in the model with search and matching frictions, respectively. We have opted against expressing the model in terms of output gaps; for our analysis in Section 4 it is more appropriate to write the model in terms of output. In contrast to standard NK models without search and matching frictions in the labor market, the level of employment $\hat{n}_t$ enters the equations. The third equation, which can be traced back to the Nash bargaining over real wages, summarizes the labor market dynamics by relating the evolution of employment to the other variables in the model.

This model reduces to the standard NK model if each household member is employed at every point in time which, among other assumptions, requires that vacancy posting costs are set to zero. Absent posting costs, κ^c assumes the value of 0, $\varpi^{y_{ss}} = 1$, and $\hat{n}_t = 0$ for all t. Equation (25) is dropped due to the lack of wage bargaining. Alternatively, the model in Ravenna and Walsh (2011) with inelastic individual labor supply emerges in the limit as ϕ approaches infinity implying $\lim_{\phi \to \infty} \theta_1 = \phi$ and $\lim_{\phi \to \infty} \gamma_2 = \phi$. Equation (25) converges to $\hat{n}_t = \hat{y}_t - \hat{a}_t$ which simply describes the production technology when hours worked are fixed. Using this result, equations (23) and (24) can be written in terms of inflation, the nominal interest rate, and employment.[11] Notice, that the search and matching model with fixed hours worked does not nest the standard NK model.

The sticky nominal wage model features NK Phillips curves for prices and wages.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa^p (\hat{w}_t - \hat{a}_t) + \hat{\theta}_{p,t} \quad (26)$$

[11] Strictly speaking, Ravenna and Walsh (2011) substitute out employment in terms of the number of job seeking workers, where the latter is proportional to the (negative) value of employment in the linearized model.

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa^w (\sigma + \phi) \left(\hat{y}_t - \frac{1+\phi}{\sigma+\phi} \hat{a}_t \right) - \kappa^w (\hat{w}_t - \hat{a}_t) \tag{27}$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) \tag{28}$$

$$\hat{w}_t = \hat{w}_{t-1} + \pi_t^w - \pi_t \tag{29}$$

Equation (29) describes the evolution of the real wage. If wages are fully flexible, the model reduces to the standard NK model since equation (27) reduces to $\hat{w}_t - \hat{a}_t = \hat{y}_t - \frac{1+\phi}{\sigma+\phi} \hat{a}_t$ and equation (26) can be written in its standard formulation.

Absent price indexation, inflation is a forward-looking phenomenon in both models and can be expressed as the discounted present value of real marginal costs, see equations (26) and (23). Although real wages are not the exclusive determinant of real marginal costs, the real wage dynamics shape the dynamics of real marginal costs importantly in both models. Thus, the dynamics of price inflation depend on the adjustment process for real wages and in turn on the impact that price inflation has on the adjustment of real wages. Monetary policy does influence the interplay between these variables.

3 Empirical Strategy

At the core of our analysis lies the idea that the policymaker can formulate multiple models that provide a good approximation to the true data-generating process given the empirical evidence against which the models are assessed. The large variety of business cycle models found in the academic literature (that all try to explain similar aspects of the data) and the diverse set of models used within central banks in practice lend support to this view.[12]

To arrive at a setting in which the policymaker considers multiple models for the purpose of policymaking, we parameterize the two models discussed in the previous section to match the same empirical evidence. Given the number of free parameters in most theoretical models, it is basically impossible to reduce the set of candidate models to a single one and obtain model certainty. This section discusses the criteria by which we judge the empirical performance of the sticky wage model and the search and matching

[12] For example, staff at the Federal Reserve derives policy recommendations for the Federal Open Market Committee from a variety of models. See the Monetary Policy Strategies section of the August 2010 Tealbook B, available under https://www.federalreserve.gov/monetarypolicy/files/FOMC20100810tealbookb20100805.pdf. In addition, several regional Reserve Banks maintain their own (publicly available DSGE) models reflecting a multitude of views about the functioning of the economy.

model.

Our approach resembles Levine, McAdam, and Pearlman (2012) in the sense that we estimate our models using the same data before exploring the optimal monetary policy for each model. Levine et al. estimate variants of the same model — the models differ with regard to parameter restrictions — using the full information Bayesian approach. By contrast, we use an impulse response function matching approach. This approach differs substantially from Svensson and Williams (2005) and Cogley and Sargent (2005). These authors estimate models under the assumption that monetary policy is set optimally given the preferences assigned to the policymaker. The central bank re-assesses over time which of the multiple models provides the best description of the economy and chooses monetary policy accordingly. At no point in time do any two models imply very similar economic outcomes contrary to our setup. The models in these papers are not designed to explain the same empirical evidence.

3.1 Estimation strategy

We estimate selected parameters of the two models using impulse response function matching introduced by Rotemberg and Woodford (1997). The empirical impulse responses against which we assess the performance of the two theoretical models are taken from Christiano, Eichenbaum, and Trabandt (2013). These authors estimate structural vector autoregressive (SVAR) models and identify shocks to monetary policy, as well as neutral and investment-specific technology shocks.

For each model, we divide the parameters into two groups: calibrated and estimated parameters. The values assigned to the first group of parameters are taken from the literature. The parameters in the second group shape the dynamics of the model importantly; with clear evidence hard to come by for these parameters, we allow the data to determine their values.

In this part of our analysis, monetary policy is assumed to follow a simple rule for the nominal interest rate as commonly found in the literature and in central bank analysis. In detail, we assume

$$i_t = \rho_R i_{t-1} + \rho_\pi \pi_t + \rho_x x_t \tag{30}$$

where π_t refers to price inflation in deviation from its long-run target value, and i_t denotes the short term nominal interest rate in deviation from steady state. The output gap is x_t. The coefficients ρ_R and ρ_π govern the degree of interest rate smoothing and the reaction of the nominal interest rate to current price inflation, respectively. In what follows, we abstract from the output gap by setting $\rho_x = 0$. When we included the output gap in the estimation, our results hardly changed.

For each model, conditional on the calibrated parameters θ^c, we search over the remaining parameters — collected in the vector θ — to minimize the distance between the impulse response functions generated from the model, denoted by $G(\theta, \theta^c)^{model}$, and the empirical impulse response functions from the SVAR in Christiano, Eichenbaum, and Trabandt (2013), denoted by G:

$$\hat{\theta} = \arg\min_\theta \left(G - G(\theta, \theta^c)^{model}\right)' \Omega^{-1} \left(G - G(\theta, \theta^c)^{model}\right). \tag{31}$$

The diagonal weighting matrix Ω is obtained from the empirical variance-covariance matrix of the empirical impulse response functions Ψ by setting all off-diagonal elements in Ψ to zero. The estimate $\hat{\theta}$ minimizes the objective function in (31). By construction, this methodology ensures that the dynamics of each model trace the dynamics in the selected empirical evidence.

Before reporting the results of the estimation, we review the values assigned to some key parameters collected in θ^c. The parameter values are recorded in Table 1. To the extent appropriate, we assign identical values to the parameters that are common across the models. In particular, we set the labor supply elasticity equal to 1/2, implying a choice of $\phi = 2$, in line with the results reported by Smets and Wouters (2007). We discuss the importance of this choice for the dynamics in the search and matching model separately below. Hours worked are assumed to be 1/3 in the steady state. The parameter ξ^p which governs the degree of nominal price rigidities is fixed at 0.75. The markup for prices is set at 20 percent in the steady state implying λ_p equal to 0.2.

In the sticky wage model, two more parameters need to be specified: the parameter governing the stickiness of nominal wages ξ^w and the steady state wage markup λ_w. As in Christiano, Eichenbaum, and Trabandt (2013), we set $\xi^w = \xi^p$, implying that wage and price contracts are updated on average once a year. The steady state wage markup is also set at 20 percent.

Parameters specific to the model with search and matching frictions are chosen as follows. The break up probability of a match with $\rho = 0.1$ implies a quarterly separation rate of 10 percent which is in line with the estimate in Shimer (2005) of 3.4 percent per month. The parameter ζ in the matching function is set at 0.54, just in the range of plausible values between 0.5 and 0.7 reported in Pissarides and Petrongolo (2001). We target a vacancy filling rate in the steady state q_{ss} of 0.7 following Ramey, den Haan, and Watson (2000), and Ravenna and Walsh (2012). The unemployment rate in the steady state is set at 0.055, the average US unemployment rate over the period from 1951Q1 to 2008Q4 reported by (Christiano, Eichenbaum, and Trabandt 2013). We assume that the costs associated with posting and filling vacancies are proportional to the number of posted vacancies. Relative to output these costs amount to η_s, which is fixed at $100 * \eta_s = 0.66$.

We estimate the remaining parameters for each model by matching the model impulse responses to a neutral technology shock to the corresponding SVAR estimates in Christiano, Eichenbaum, and Trabandt (2013). We include the first 15 periods after the shock. While we fix the persistence of the technology shock at $\rho_a = 0.9999$, we estimate the standard deviation of the shock σ_a. Furthermore, for each model we estimate the coefficients in the interest rate rule, ρ_R and ρ_π, the degree of (internal) consumption habits h, and the degree of price indexation ι^p. In the search and matching model, we also estimate the replacement ratio r^u. Finally, we estimate two versions of the sticky wage model; in the first one we abstract from wages indexation, i.e., ι^ω is fixed at 0, and in the second one we estimate the degree of wage indexation. For the sticky wage model, the estimation includes the impulse response functions for output, inflation, the short-term interest rate as measured by the federal funds rate, hours worked, real wages, and consumption. In the case of the search and matching model, we also include the responses of the unemployment rate, vacancies, and the job finding rate.

Table 2 summarizes the estimates. Figure 1 shows the resulting impulse responses for the two models absent wage indexation in the sticky wage model and Figure 2 plots the responses when the degree of wage estimation is estimated. In both figures, the theoretical models match the empirical responses well; with the exception of hours worked, the model responses lie within the confidence bands of the empirical responses and the responses are reasonably close to the SVAR point estimates and to each other. The sticky wage model with indexation, estimated at $\iota^\omega = 1$, provides a better fit to the data than the model without wage indexation according to the value of our criterion function, see bottom of Table 2. Most of the difference in fit stems from the model's implications for hours worked. However, some authors have expressed skepticism regarding the presence of wage indexation in the data, see Levine, McAdam, and Pearlman (2012) and Christiano, Eichenbaum, and Trabandt (2013).

The estimates for the coefficients in the policy rule, the variance of the technology shock, price indexation, and consumption habits are almost identical across models. The estimated policy rule features a high degree of interest rate smoothing and the implied long-run response of the interest rate to inflation is just strong enough to satisfy the Taylor principle, e.g., for the search and matching model this value is $1.0003 = (1 - 0.8555)/0.14445$. The estimated simple interest rate rules are close to identical across models. For all model specifications, price indexation is estimated to be zero. Overall, our estimates resemble those in Christiano, Eichenbaum, and Trabandt (2013) despite the greater simplicity of our models — Christiano et al. include investment, capacity utilization, and the relative price of investment in the set of impulse responses and they require their models to also match the empirical responses to monetary policy and investment-specific technology shocks. Two important differences emerge, however. First, our models do not rely on internal consumption habits to match the empirical responses ($h = 0$), whereas Christiano et al. estimate h to lie between 0.7 and 0.8. This result in Christiano et al. appears to stem from their inclusion of the monetary policy shock in the estimation. Their (recursively identified) monetary policy shock induces a pronounced hump-shaped response in consumption, a feature that the consumption response does not show for the neutral or for the investment-specific shock. Habit persistence is key to match the consumption response to the monetary policy shock. When we fix h at a strictly positive value and estimate the model again, the remaining parameters reported in Table 2 change slightly while the overall fit of the impulse responses remains of similar quality. Unless noted otherwise, we abstract from consumption habits from there on. Second, we estimate the replacement ratio r^u at 0.5345, which is well below the implausibly high estimate in Christiano, Eichenbaum, and Trabandt (2013) for the search and matching model with Nash bargaining. We elaborate on the role of the replacement ratio next.

3.2 The elasticity of labor market tightness with respect to shocks

The responses of unemployment and vacancies are important dimensions to judge the empirical performance of the search and matching model. The unemployment rate (and thus the number of job seekers u_t) drops significantly after rising initially and vacancies v_t increase strongly over the medium term. Both in the data and the model the directions and the magnitudes of these responses imply a strong response of labor market tightness (the ratio of unfilled vacancies to job seekers).

As shown in Appendix B.1, labor market tightness $\hat{\theta}_t$ (expressed in log deviation from steady state) is approximately proportional to (the log-deviations from steady state of) the marginal product of labor, hours worked, and real marginal costs in our model:

$$\begin{aligned}\hat{\theta}_t &= \hat{v}_t - \hat{u}_t \\ &\approx \frac{1}{\Upsilon} \frac{\frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss}}{\left[\left(\frac{\phi}{1+\phi} - r^u\right) mpl_{ss} h_{ss} mc_{ss} + r^u \left(1-(1-\rho)\beta\right) \frac{\kappa^v}{q_{ss}}\right]} \left(\widehat{mpl}_t + \hat{h}_t + \widehat{mc}_t\right) \quad (32)\end{aligned}$$

where

$$\Upsilon = \zeta + \frac{(1-\rho)\beta\xi q_{ss}\theta_{ss}\left(1-\zeta\right)}{\left[1-(1-\rho)\beta\left(1-\xi q_{ss}\theta_{ss}\right)\right]}. \quad (33)$$

Υ lies in the interval $[\zeta, 1]$, where ζ is often set around 0.5 (in our case 0.54).

Abstracting from the disutility of working for employed workers (i.e., setting ϕ at an infinitely large number), Shimer (2005) argues that standard search and matching models cannot produce the strong response of labor market tightness relative to the movements in the marginal product of labor compared to the empirical evidence under plausible parameter choices, in particular for the replacement ratio r^u. According to Shimer, a strongly pro-cyclical real wage dampens the responses of vacancies and unemployment resulting in a much muted response of labor market tightness vis-a-vis the data.[13]

Although Hall (2005), Hagedorn and Manovskii (2008), Hall and Milgrom (2008), Pissarides (2009), and Christiano, Eichenbaum, and Trabandt (2013) among others offer approaches to resolve this issue, there is no paper that studies the optimal monetary policy

[13] For our parameterization, the steady state values of the marginal product of labor mpl_{ss} and marginal costs mc_{ss} are 1, and hours worked h_{ss} are 1/3, implying $h_{ss} mpl_{ss} mc_{ss} = 1/3$. With the term $(1-(1-\rho)\beta)\frac{\kappa^v}{q_{ss}}$ assuming the value 0.0024, the elasticity of labor market tightness can be raised to its value in the data by choosing r^u sufficiently close to 1. In a setting similar to ours, Christiano, Eichenbaum, and Trabandt (2013) estimate r^u to be 0.88.

for a NK model with search and matching frictions with empirically plausible labor market responses. Our framework avoids the criticism in Shimer (2005) by modeling the disutility from working explicitly. With a labor supply elasticity of 0.5, i.e., $\phi = 2$, the required value for r^u in order to match the empirical evidence on unemployment, vacancies, and labor market tightness drops from almost 1 to near 0.5.[14]

3.3 Additional shocks

In addition to technology shocks, our model features markup shocks. Unfortunately, we are not aware of a broadly accepted scheme to identify markup shocks using SVAR analysis. Without resorting to full information estimation of the two models as in Smets and Wouters (2007), we proceed by assuming that each economy is subject to purely transitory markup shocks. The standard deviation of the markup shock is set at 0.0135 in the sticky wage model. The standard deviation of the markup shock in the search and matching model of 0.0104 minimizes the distance between the impulse responses for the markup shocks in the two models given the remaining parameters in Tables 1 and 2. The smaller value of the standard deviation in the search and matching model reflects the stronger impact of an equal-sized markup shock on output and inflation in the search and matching model compared to the sticky wage model.

4 Optimal policy and robustness

In matching our two models to the empirical evidence, we assumed that monetary policy follows a simple interest rate rule similar to Taylor (1993) and estimated the coefficients of the rule. While these estimated rules provide a good fit to actual monetary policy, they do not necessarily implement the optimal monetary policy.

Although the search and matching model and the sticky wage model imply similar responses to shocks, in particular for price and wage inflation, under the estimated rules we show that this is no longer the case when monetary policy is set optimally in the two models. Furthermore, implementing the policy that is optimal in the search and matching model in the sticky wage model leads to responses that are far from optimal under sticky nominal wages. The same issue arises if the roles are reversed. We show

[14] Hall and Milgrom (2008) suggest replacing the assumption of Nash bargaining by an alternating offer bargaining game. As shown in Bodenstein, Kamber, and Thoenissen (2016), this alternative approach has similar empirical implications as ours.

this lack of robustness of the optimal policy with the help of optimal targeting rules. In the following, when referring to the sticky wage model we abstract from wage indexation unless stated otherwise.

4.1 Optimal policy and optimal targeting rule

Svensson and Woodford (2004), Giannoni and Woodford (2003, 2016) advocate for the use of optimal targeting rules to characterize the optimal monetary policy. The optimal targeting rule specifies the variables — including the relative importance and the dynamic structure of each variable — in a single target criterion that seeks to implement the optimal monetary policy.

Optimal targeting rules can be computed for any preferences assigned to the policymaker. However, we adopt the approach in Woodford (2003) to have the preferences of the policymaker coincide with those of the representative agent in the model. Therefore, obtainaing optimal targeting rules in our settings requires to: (1) derive the objective function of the policymaker as a purely quadratic approximation to the preferences of the representative household;[15] (2) obtain the first order conditions associated with the policymaker's problem of optimizing the (quadratic) objective function subject to the (linear) equations that describe the behavior of the private sector using the Lagrangian approach; (3) combine the first order conditions to obtain a single equation without Lagrange multipliers; this targeting rule describes the relationship between the endogenous and exogenous variables under the optimal policy. Importantly, the evolution of the economy that is consistent with the targeting rule is unique.

The optimal targeting rule implements the optimal monetary policy in the model from which it is derived, in contrast to simple instrument rules with optimally chosen coefficients. However, similar to instrument rules, the optimal targeting rule is expressed in terms of economically relevant model variables only; instrument rules prescribe how to adjust the policy instrument (such as the short term interest rate) in response to variables such as inflation, output, etc., and targeting rules describe how to adjust a target variable (for example, price inflation) to output, wage inflation and other variables. Finally, the evolution of the optimal target criterion does not depend on the policy instrument of the central bank itself. Optimal targeting rules are therefore ideally suited to investigate the

[15] We follow Woodford (1999) and Benigno and Woodford (2012) in adopting the concept of "optimality from the timeless perspective" — a necessary assumption to obtain the correct linear quadratic approximation to our (nonlinear) model.

robustness of the optimal policy of one model in a different model.

The optimal targeting rule for the NK model with sticky wages is easily obtained from the linear quadratic approximation of the original model. Employing results from Woodford (2003) for the policymaker's objective function and the linear approximation of the model's structural equations, the policymaker's problem is to

$$\min_{\{\pi_t,\pi_t^w,\hat{y}_t,i_t,\hat{w}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{sw} \tag{34}$$

s.t. equations (26) to (29) and suitably specified pre-commitments for period 0.[16]

Absent consumption habits, price and wage indexation, the loss function consistent with the second order approximation of household preferences satisfies

$$\mathcal{L}_t^{sw} = \frac{\sigma+\phi}{2}\left(\hat{y}_t - \frac{1+\phi}{\sigma+\phi}\hat{a}_t\right)^2 + \frac{1+\theta^p}{2\theta^p\kappa^p}\pi_t^2 + \frac{1+\theta^w}{2\theta^w\kappa^w}(\pi_t^w)^2. \tag{35}$$

Giannoni and Woodford (2003) show that the first order conditions associated with the problem in (34) can be recombined to obtain the optimal targeting rule

$$\begin{aligned} -\chi_1\pi_t &= \chi_2\left(\pi_{t+1}^w - \pi_t^w\right) + \chi_3\pi_t^w + \chi_4\left(\pi_t^w - \pi_{t-1}^w\right) + \chi_5\left[(\hat{y}_{t+1}-\hat{y}_t) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_{t+1}-\hat{a}_t)\right] \\ &+\chi_6\left[(\hat{y}_t-\hat{y}_{t-1}) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_t-\hat{a}_{t-1})\right] + \chi_7\left[(\hat{y}_{t-1}-\hat{y}_{t-2}) - \frac{1+\phi}{\sigma+\phi}(\hat{a}_{t-1}-\hat{a}_{t-2})\right]. \end{aligned} \tag{36}$$

We continue to express the rule in terms of output and the underlying shocks instead of the output gap ($x_t = \hat{y}_t - \frac{1+\phi}{\sigma+\phi}\hat{a}_t$). See Appendix D for details of the derivation. Absent sticky wages, the rule in equation (36) suggests lowering the target value for inflation in the current period below its long run target when the growth rate of the output gap is positive as during the recovery from a shock that caused a negative output gap ($\chi_1 > 0$, $\chi_6 > 0$, all other coefficients are zero in this case). If wages are sticky and growth in the output gap is positive, wage inflation below its steady state value, ceteris paribus, calls for bringing inflation closer to its long-run target value than under flexible wages as $\chi_3 > 0$.

Thus far, derivations of the optimal targeting rules in models with search and matching frictions are absent from the literature. Blanchard and Galí (2010), Thomas (2008), and Ravenna and Walsh (2011) derive purely quadratic objectives for the policymaker from household preferences under the assumption that the search and matching process does not induce inefficiencies as in Hosios (1990). None of these papers derives the implied optimal targeting rule. Furthermore, if the Hosios condition is not imposed, even the first step of obtaining an explicit second order approximation to the preferences of the representative household that serves as the policymaker's model-consistent objective is missing in the literature.

To derive a purely quadratic objective for the policymaker in the presence of a distorted steady state, we employ the numerical approach described in Bodenstein, Guerrieri, and LaBriola (2014) which is consistent with the theoretical results in Benigno and Woodford (2012). Appendix C shows that the policymaker's (period) loss function consistent with a second order approximation to the preferences of the representative household can be written as

$$\begin{aligned}\mathcal{L}_t^{s\&m} &= P_{\pi,\pi}\pi_t^2 + P_{y,y}\hat{y}_t^2 + P_{n,n}\hat{n}_t^2 + P_{n^-,n^-}\hat{n}_{t-1}^2 + P_{y,n}\hat{n}_t\hat{y}_t + P_{y,n^-}\hat{y}_t\hat{n}_{t-1} \\ &\quad + P_{n,n^-}\hat{n}_t\hat{n}_{t-1} + P_{n,a}\hat{n}_t\hat{a}_t + P_{n,p}\hat{n}_t\hat{\theta}_{p,t} + P_{y,a}\hat{y}_t\hat{a}_t + P_{y,p}\hat{y}_t\hat{\theta}_{p,t}. \end{aligned} \tag{37}$$

This formulation of the loss function is already simplified to include those variables only that enter the linear model in equations (23)-(25). We cannot obtain closed form expressions for the composite coefficients in (37), but our approach provides numerical values based on the underlying deep parameters of the model. The (linear-quadratic) problem of the policymaker is

$$\min_{\{\pi_t, i_t, \hat{n}_t, \hat{y}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t^{s\&m} \tag{38}$$

$s.t.$ equations (23) to (25) and suitably specified pre-commitments for period 0.

Rearranging the first order conditions associated with the optimization problem in (38) delivers the optimal targeting rule in the search and matching model

$$\begin{aligned}&\varpi_1\hat{n}_t + \varpi_2\hat{n}_{t-1} + \varpi_3\hat{n}_{t+1} + \varpi_4\hat{y}_t + \varpi_5\hat{y}_{t+1} + \varpi_6\hat{a}_t + \varpi_7\hat{\theta}_{p,t} + \varpi_8\pi_t + \varpi_9\pi_{t+1} + \varpi_{10}\hat{P}_{t-1} \\ &+\varpi_{11}\hat{y}_t^{WA} + \varpi_{12}\hat{n}_t^{WA} + \varpi_{13}\hat{a}_t^{WA} + \varpi_{14}\hat{\theta}_{p,t}^{WA} + \varpi_{15}\hat{P}_t^{WA} = 0\end{aligned} \tag{39}$$

where we define

$$\pi_t = \hat{P}_t - \hat{P}_{t-1} \tag{40}$$

$$\hat{y}_t^{WA} = \beta_\delta \hat{y}_{t-1}^{WA} + \hat{y}_t \tag{41}$$

$$\hat{n}_t^{WA} = \beta_\delta \hat{n}_{t-1}^{WA} + \hat{n}_t \quad (42)$$

$$\hat{P}_t^{WA} = \beta_\delta \hat{P}_{t-1}^{WA} + \hat{P}_t. \quad (43)$$

$$\hat{a}_t^{WA} = \beta_\delta \hat{a}_{t-1}^{WA} + \hat{a}_t \quad (44)$$

$$\hat{\theta}_{p,t}^{WA} = \beta_\delta \hat{\theta}_{p,t-1}^{WA} + \hat{\theta}_{p,t} \quad (45)$$

In the steady state, the price level grows with the steady state inflation rate. The term $\hat{P}_t$ denotes deviations of the price level from this growth path. Compared to the sticky wage model, the targeting rule in the search and matching model involves weighted infinite-moving averages of output, employment, the price level, and the shocks. The presence of the markup shock $\hat{\theta}_{p,t}$ in the targeting rule is solely due to our decision not to impose the efficiency condition by Hosios (1990).[17] For the parameters in Table 1 and 2, the weights ϖ_j with $j = 1, ..., 15$ are such that the optimal targeting rule from the standard NK model (without search and matching frictions) is a close approximation to the rule in equation (39) as discussed in more details in the companion paper Bodenstein and Zhao (2016).

4.2 Robustness of optimal targeting rules

The optimal targeting rules in equations (36) and (39) are stated in terms of variables that are common across models. We can therefore compute the responses of prices and quantities in the sticky wage model under the rule in equation (39) and in the search and matching model under the rule in equation (36). We start by comparing the impulse responses in the search and matching model under the two rules. Figure 3 depicts the case of the technology shock $\hat{a}_t$, and Figure 4 shows the case of the price markup shock $\hat{\theta}_{p,t}$.

Under the technology shock, the optimal monetary policy as implemented by the targeting rule (39) calls for almost full stabilization of price inflation. No meaningful trade-offs arise as the welfare relevant gaps move in the same direction: the technology shock exerts downward pressure to prices, and upward pressure on output and employment with the expansions being held back by sticky nominal prices. An interest rate cut reduces the downward pressure on prices and speeds up the expansion in output and

[17] In the sticky wage model the markup shock does not enter equation (36) since the steady state is assumed to be efficient; otherwise the markup shock would appear in the targeting rule as well. See also Benigno and Woodford (2005).

employment. As a result, the real variables follow closely their paths in a real economy without sticky prices. These findings are reminiscent of the standard NK model with flexible wages in which the optimal monetary policy fully stabilizes inflation and closes the output gap.

Notably, the labor market adjusts almost instantaneously to the shock in sharp contrast to the empirical responses in Figure 1. Under the optimal policy the real wage adjusts swiftly facilitated by a pronounced spurt in wage inflation. The movements in wages reflect the persistent jump in the marginal value of employment to the firm that gives rise to the front-loaded response in vacancies and the fall in unemployment.

In contrast to equation (39), the rule given in equation (36) places greater emphasis on stabilizing wage inflation and less emphasis on price inflation. With monetary policy holding nominal wages basically constant, but the persistent rise in technology pressuring real wages to rise, price inflation must fall below its target value to facilitate at least gradual adjustment in the real wage. As firms and households cannot reap all the benefits of higher technology and higher real wages immediately, vacancy postings, employment, and unemployment display inertia relative to the optimal responses. Adjustment in output and consumption is consequently delayed, as well.

In the case of the markup shock, similar differences emerge between the two policy rules in the search and matching model. With the exception of price inflation, all other variables react more strongly to the shock under the optimal policy, equation (39). As the markup shock induces a trade-off between variables, price inflation is not fully stabilized under the optimal policy to temper the fluctuations in the other variables. Again, when the targeting rule derived from the sticky wage model is imposed instead, i.e., the rule in equation (36), wage inflation is almost fully stabilized at the expense of higher price inflation. The responses of all other variables are greatly muted compared to the optimal policy case.

The lack of robustness of the targeting rules across models also applies to the model with sticky wages. Figures 5 and 6 plot the responses in the sticky wages model to technology and markup shocks, respectively, for the two targeting rules. The optimal policy under sticky nominal wages, implemented through equation (36), stabilizes wage inflation in response to both shocks. This policy avoids welfare-costly wage dispersion, whereas price inflation induces movements in the real wage that in turn facilitate the adjustment process for all other variables under the optimal policy. By overly stabilizing price inflation, the targeting rule derived in the search and matching model (39) allows more wage inflation than is optimal and in turn causes hours worked, output, and consumption, in particular, to exceed the optimal responses noticeably.

To sum up, the targeting rule (39), which is optimal in the search and matching model, favours stabilizing prices over stabilizing wages irrespective of the model in which the rule is implemented. The targeting rule (36), which is optimal in the sticky wage model, favours stabilizing wages over stabilizing prices irrespective of the model under consideration. Exchanging targeting rules between the models induces welfare losses that are orders of magnitudes larger than the welfare costs of business cycles in Lucas (2003). For the sticky wage model the welfare loss (measured in CEV) is considerably higher than for the search and matching model (1.3033 versus 0.1133) reflecting the high welfare costs associated with even minor relative wage differences in the sticky wage model.

4.3 Sensitivity

The lack of robustness of the optimal targeting rules may depend on our modelling choices. We investigate two avenues to explore the sensitivity of this result: (i) wage indexation in the sticky wage model, and (ii) the preferences assigned to the policymaker.

The estimation results in Table 2 suggest that the empirical fit of the sticky wage model improves if we allow for indexation of wages to past inflation. With full wage indexation, the focus of optimal monetary policy in the sticky wage model shifts from smoothing wage inflation to smoothing the difference between wage inflation and lagged price inflation, i.e., $\pi_t^w - \pi_{t-1}$. This change in focus of the optimal policy is also reflected in the optimal targeting rule derived for the sticky wage model with $\iota^\omega = 1$.

Figure 7 plots selected impulse responses to a markup shock when the sticky wage model features full wage indexation and we repeat the previous exercise of comparing the outcomes in the search and matching model and the sticky wage model (now with $\iota^\omega = 1$) under the optimal targeting rules derived in the two models, respectively. Under full wage indexation, the optimal monetary policy in the sticky wage model refrains from stabilizing wage inflation; to reduce welfare-costly dispersion in the nominal wage, the central bank smooths the term $\pi_t^w - \pi_{t-1}$. Under the markup shock, the decline in the real wage is still engineered by raising inflation in the impact period. Yet, the rise in price inflation this period pushes up nominal wages in the subsequent period through indexation which in turn offsets most of the decline in the real wage. To compensate for this effect, price inflation rises by more in the impact period under the optimal policy in the model with indexation than absent indexation. Turning to the optimal targeting rule derived in the search and matching model, this rule with its focus on reducing price inflation induces even bigger welfare losses (measured as CEV) in the sticky wage model with full indexation than in the model without indexation (now 1.9728 instead of 1.3033), confirming the lack of robustness of the optimal targeting rules.

For the search and matching model, wage indexation in the sticky wage model only impacts the responses under the optimal targeting rule from the sticky wage model relative to the previous discussion; the differences between the two targeting rules are mostly quantitative in nature. Given the modified focus of the new targeting rule from the sticky wage model, wage inflation is not stabilized as forcefully as in Figure 4. Yet, since nominal wages in period t move to offset past inflation, the downward adjustment in the real wage demands even larger movements in inflation than under the no-indexation targeting rule. Thus, the overall welfare loss in the search and matching model (measured as CEV) rises (now 0.1680 instead of 0.1133 for $\iota^{\omega} = 0$).

Our two models differ with regard to the details of the labor market and, as a result of our choice to assign to the policymaker the preferences of the representative household, the models also differ with regard to the objective function assigned to the central bank. To glean a better understanding of the role of these differences across models, we consider the case that the policymaker's preferences are given by the simple quadratic loss function of the type $\mathcal{L}_t^{sql} = \pi_t^2 + \lambda_x x_t^2$ irrespective of the underlying model. This objective function is commonly used in the literature and policy analysis. Examples are Levin, Wieland, and Williams (2003), Svensson and Williams (2005), and Debortoli, Kim, Lindé, and Nunes (2015). The parameter λ_x governs the relative importance of stabilizing price inflation versus the output gap. We consider two parameterizations of λ_x: under the first one, $\lambda_x = 0.0429$, the policymaker places high emphasis on price inflation similar to the policymaker with preferences $\mathcal{L}^{s\&m}$; under the second one, $\lambda_x = 1$, the emphasis on price inflation is low as for a policymaker with preferences $\mathcal{L}^{sw}$.[18] Figure 8 shows the impulse responses under the targeting rules derived from the simple loss functions.

In the search and matching model, the optimal policy consistent with preferences $\mathcal{L}^{sql}$ with $\lambda_x = 0.0429$, resembles the optimal policy derived under preferences $\mathcal{L}^{s\&m}$ — the

[18] The choice $\lambda_x = 0.0429$ is consistent with the weight on the output gap in the loss function derived for the standard NK model with flexible wages under the parameters in Tables 1 and 2. The alternative specification of $\lambda_x = 1$ is popular in the literature.

two top rows of panels in the figure. However, with the exception of price inflation all variables react by less to the markup shock than in Figure 4, indicating that under our parameterization of $\mathcal{L}^{sql}$ the policymaker prefers price inflation to bear more of the burden of adjustment than in our original case. When imposing onto the search and matching model the optimal targeting rule derived in the sticky wage model under preferences $\mathcal{L}^{sql}$, the same qualitative differences emerge as in Figure 4 despite the fact that the policymaker's preferences are now constant across models. The optimal targeting rule is a function of both the policymaker's preferences and the underlying economic model. Similarly, in the sticky wage model the gaps between the impulse responses under the two targeting rules derived for preferences $\mathcal{L}^{sql}$ remain large albeit smaller than in Figure 6. Once again, the optimal targeting rules are not robust across models.

However, if the policymaker assigns even lower relative importance to price inflation, the optimal targeting rules *are* robust. The lower two rows of panels in Figure 8 show the impulse responses for $\lambda_x = 1$. In both the search and matching model and the sticky wage model, the gaps between the impulse responses generated by the optimal targeting rules derived for $\lambda_x = 1$ in the two models are minor. It is important to realise, though, that the robustness of the optimal targeting rules under $\lambda_x = 1$ only applies from the viewpoint of the policymaker with preferences $\mathcal{L}^{sql} = \pi_t^2 + x_t^2$. For the representative households these policies are suboptimal as household preferences continue to be given by $\mathcal{L}^{s\&m}$ and $\mathcal{L}^{sw}$, respectively. The robustness of optimal targeting rules is definitively sensitive to the preferences assigned to the policymaker. As in Levin and Williams (2003) obtaining robustness of optimal targeting rules across models requires a reduced focus on price stability.

5 Robust policy

Our findings pose a policy dilemma: we have two models that by design match the selected empirical evidence under estimated simple interest rate rules reasonably well, but the two models recommend conflicting policy actions from the optimal policy perspective. How should a policymaker conduct monetary policy if the policymaker considers both models to be good approximations to the true data-generating process?

It is straightforward to include more than two reference models into the framework. One such candidate is the model by Gertler and Trigari (2009) which merges the ideas of the search and matching framework with those of nominal rigidities in wage setting.[19] We could also include reference models that have the same form of our two models but differ with respect to the parameter choices like the sticky wage model with full wage indexation.

5.1 Methodology

To analyze optimal monetary policy under model uncertainty with multiple reference models we follow the approach in Levin and Williams (2003) and Levin, Wieland, and Williams (2003). This approach requires us to specify the objective function of the policymaker, the economic models to be considered by the policymaker (including a probability distribution over these models), and a flexible policy rule to summarize the optimal policy.

5.1.1 Objective function of the policymaker and models

There are two dimensions to the objective function. First, we need to decide on the preferences of the policymaker over the outcomes within a given model; second, we need to formalize the policymaker's strategy for dealing with model uncertainty over the two models.

Previous works of model uncertainty tend to separate discussing the policymaker's preferences from the underlying model; often these preferences are assumed to be represented by a simple quadratic loss function that punishes deviations of inflation and the output gap from their respective long-run target values and the preferences do not differ across models. We assign model-consistent preferences to the policymaker. For each model, the preferences of the policymaker over model outcomes coincide with those of the representative household in that model. By construction, the policymaker's preferences differ across the reference models in our framework. In the following, we denote the policymaker's preferences over outcomes in the search and matching model by the function $\mathcal{L}^{s\&m}(\Theta)$ and in the sticky wage model by the function $\mathcal{L}^{sw}(\Theta)$, see equations (37) and (35), respectively.

We consider two strategies to cope with model uncertainty: the Bayesian and the minmax approach. Under the Bayesian approach, the policymaker's objective function

[19] At least for the calibration in Thomas (2008), the optimal policy recommendations derived from this hybrid framework with regard to inflation stabilization resemble those of the standard sticky wage model. In the interest of keeping our setup simple we choose not to include a model of this sort.

weighs the various models according to their assigned probabilities

$$\mathcal{L}^{Bayesian} = \omega * \mathcal{L}^{s\&m}(\Theta) + (1-\omega) * \mathcal{L}^{sw}(\Theta) \tag{46}$$

where ω is the weight assigned to the search and matching model and $1-\omega$ is the weight on the sticky wage model.

The Bayesian strategy can be interpreted literally as the case of a single policymaker assigning a probability distribution over the reference models based on statistical analysis. For example, Levine, McAdam, and Pearlman (2012) translate posterior odds ratios of models estimated with full information Bayesian techniques into the policymaker's probability distribution over models. The policymaker then engages into model averaging. In this sense, the policymaker's model is a weighted average across the reference models. An alternative interpretation is related to decision-making in committees. Each member of the committee selects a single model that reflects her/his views over the economy. The optimal policy under uncertainty is not optimal from any individual member's point of view, but it produces outcomes that might be acceptable to all members.

When the policymaker pursues the minmax strategy, the objective is to minimize the maximum welfare loss across the search and matching model and the sticky wage model

$$\mathcal{L}^{minmax} = \max\left\{\mathcal{L}^{s\&m}(\Theta), \mathcal{L}^{sw}(\Theta)\right\}. \tag{47}$$

This policymaker is strongly concerned with avoiding the worst case scenario of setting a policy that could result in large welfare losses under any circumstances.

5.1.2 Formulating policy

In computing the optimal monetary policy under model uncertainty we assume that monetary policy follows a simple instrument rule of the type

$$i_t = \rho_R i_{t-1} + \rho_\pi \pi_t + \rho_\pi^w \pi_t^w + \rho_x x_t. \tag{48}$$

We collect the coefficients of the rule in the vector $\Theta = \left\{\rho_R, \rho_\pi, \rho_\pi^w, \rho_x\right\}$. According to the rule, the nominal interest rate is adjusted in response to movements in price and wage inflation, as well as the output gap. Furthermore, the rule allows for interest rate inertia. We search for the (non-negative) values of the parameters in Θ that minimize the welfare loss of the policymaker under the given objective.[20]

We restrict attention to simple rules in this section in order to maintain transparency. Adapting the ideas in Svensson and Williams (2005), it would be possible to compute the optimal policy under model uncertainty using the tools of the optimal control literature. However, characterizing the optimal policy in terms of a targeting rule turns difficult under model uncertainty with multiple reference models; the resulting targeting rule is hard to analyze. After all, interpreting the targeting rule for the search and matching model in equation (39) is already cumbersome, see Bodenstein and Zhao (2016) for details. Although optimal targeting rules obtain higher welfare outcomes than optimal simple rules, simple rules can provide high-quality approximations to the optimal policy even if the number of variables included in the rule is small. We confirm that without model uncertainty the simple rule (48) can indeed approximate the optimal policy with a high degree of accuracy.

5.2 Monetary policy rules under model uncertainty

Table 3 reports the optimal simple rules under the benchmark parameterization of the search and matching model and the sticky wage model. For lack of clear empirical guidance, we consider multiple probability distributions with ω, the probability that the policymaker assigns to the search and matching model being the true data-generating process, ranging from 0 to 1 for the Bayesian approach. We refer to the optimal simple rule associated with a given probability distribution as the "ω-optimal simple rule." [21] The policymaker's probability distribution over models is irrelevant under the minmax strategy. The table also reports the welfare implications in both models under each policy rule. Welfare is expressed in terms of the loss functions and consumption equivalent variations (CEV). Finally, we report the welfare costs of implementing in the model under consideration the targeting rule that is optimal in the other model (exchanging optimal targeting rules).

Under the Bayesian strategy, we distinguish three regions for the probability ω: low

[20] For a given parameterization of the rule, we compute the unconditional welfare loss. Following Benigno and Woodford (2012) we include a correction term to account for the fact that the rule may violate the pre-commitment conditions imposed in deriving the loss functions. Rules that lead to equilibrium indeterminacy are discarded.

[21] Although the empirical evidence discussed in Section 3 can be viewed as favouring the search and matching model over the sticky wage model if wages are not indexed, the sticky wage model with full indexation appears to have stronger support in the data than the search and matching model. Absent formal model comparison — which is complicated by the fact that estimation of the three models does not employ same data series — and the predominance of the sticky wage approach in the DSGE models used at central banks, exploring a wide range of values for ω seems well-advised.

($\omega \leq 0.2$), intermediate range ($0.3 \leq \omega \leq 0.8$), and high ($\omega \geq 0.9$). The optimal simple rule varies distinctly across these regions. In the first region with little probability weight on the search and matching model, the nominal interest rate responds primarily to wage inflation in line with the optimal policy prescriptions of the sticky wage model. In the second region, the rule responds to wage and price inflation with the coefficients assigned to the two variables being of similar magnitude. It is only in the third region that the optimal rule displays significant interest rate inertia. The coefficient on wage inflation basically drops to zero whereas the nominal interest rate responds to price inflation. With the policymaker assigning a high probability to the search and matching model, the importance of wage inflation stabilization fades. Consequently, in the sticky wage model the welfare loss (relative to the optimal monetary policy in that model) under the ω-optimal simple rule is larger for higher values of ω and the welfare loss in the search and matching model is reduced.

While the implications of the policy rules in the first and third region of the table are easily understood and rationalized simply through the magnitude of the weight ω, the rules in the intermediate second region are harder to understand. We thus plot the impulse responses of output, price and wage inflation in the two models to both the technology and the markup shock for the optimal simple rules derived under $\omega = 0, 0.2, 0.3, 0.8, 0.9, 1$ in Figure 9. In the sticky wage model, the ω-optimal simple rules with $\omega < 0.9$, induce impulse responses (bottom two rows of panels) that are reasonably close to those under $\omega = 0$ (the optimal simple rule if the policymaker is certain about the sticky wage model being the true data-generating process). For the search and matching model, ω-optimal simple rules with $\omega < 0.9$ induce responses that differ noticeably from those under $\omega = 1$ (the optimal simple rule if the policymaker is certain about the search and matching model being the true data-generating process).

This conclusion is supported by two more pieces of evidence. First, the welfare losses in Table 3 induced by the ω-optimal simple rule change noticeably as ω increases from 0.8 to 0.9. The CEV value for the sticky wage model goes from negligible to 0.2. While the welfare losses in the search and matching model are generally small, the CEV value is practically zero under the ω-optimal simple rule for $\omega = 0.9$. Second, we compute the Euclidean distances between the impulse response functions. If the optimal simple rules are examined in the search and matching model, we compute the Euclidean distance between the impulse responses under each ω-optimal simple rule and the ω-optimal simple rule with $\omega = 1$. The Euclidean distances in the sticky wage model are computed relative to the impulse responses under the ω-optimal rule for $\omega = 0$. Again, unless the probability weight assigned to the search and matching model is 0.9 or above, the ω-optimal simple rule bears close resemblance with the optimal targeting rule *derived in the sticky nominal wage model*. Policies that are (close to) optimal in the search and matching model absent model uncertainty are not robust to the (small) possibility that the sticky wages model is the true data-generating process.

The reason to the apparent bias of the optimal policy under model uncertainty towards the sticky wage model lies in the high welfare costs associated with even minor relative wage differences in the sticky wage model. The desire to avoid bad economic outcomes caused by bad monetary policy is even more explicit when the policymaker adopts a minmax strategy. In this case, the optimal simple rule coincides with the ω-optimal simple rule for $\omega = 0$, which in turn mimics the optimal targeting rule derived in the sticky wage model.[22]

5.3 Sensitivity of results

5.3.1 Functional form of the policy rule

In principle, the policy rule in equation (48) allows the policymaker to respond to the lagged value of the nominal interest rate, price and wage inflation, and the output gap. However, each variable is assigned the value of zero for some ω; the patterns of zeroes define the three distinct regions of the ω-optimal simple rules in Table 3 for the Bayesian approach. To assess the sensitivity of our findings to the functional form of the policy rule, Table 4 reports optimal simple rules that, in comparison to (48), are restricted not to respond to either the lagged interest rate, the output gap, price inflation, or wage inflation, respectively.[23]

Absent interest rate smoothing (Case I in Table 4), the optimal simple rule changes only for $\omega \geq 0.9$ compared to Table 3. The response coefficient for price inflation becomes very large to compensate for the lack of interest rate smoothing in the rule, but overall

[22] The ω-optimal simple rule with $\omega = 0$ is very close to but not identical to the optimal targeting rule derived in the sticky wage model, as the functional form of the policy rule in equation (48) is not quite flexible enough.

[23] This analysis also sheds light on the optimality of the computed rules. The presence of three distinct parameter regions under the Bayesian approach suggests the existence of multiple local optima. In computing restricted optimal simple rules we retrieve additional confirmation regarding the ω-optimal simple rules reported in Table 4 are indeed globally optimal.

welfare and welfare in the search and matching model deteriorate nevertheless. In its eagerness to fight price inflation, the rule for $\omega = 1$ and $\rho_R = 0$ is particularly unattractive, as it induces welfare losses in the sticky wage model that by far exceed the corresponding loss in Table 4.

Eliminating the output gap from the list of response variables (Case II) affects the computations of the optimal simple rules only for $\omega \leq 0.2$. These restricted rules respond to wage inflation by more than in Table 3 — the optimizer reaches the upper bound of 100 — where the ω-optimal simple rule responded importantly to the output gap for $\omega \leq 0.2$. The overall welfare loss is higher mostly because the restricted rules perform worse in the sticky wage model.

More dramatic changes in the optimal simple rules appear if the rules are restricted not to respond to price inflation or wage inflation (Case III). Setting $\rho_\pi = 0$ leads to higher response coefficients for wage inflation (and the output gap or interest rate smoothing depending on the value of ω). The deterioration in overall welfare is borne by the search and matching model; welfare in the sticky wage model improves for most values of ω and never declines.

Finally, when eliminating the policymaker's ability to respond to wage inflation directly, welfare losses increase in both the sticky wage and the search and matching model for most values of ω (Case IV). The form of the simple rule in this final case coincide with the specification adopted in our estimation. Even more so, under $\omega = 0.8$ and $\omega = 0.9$, the restricted optimal simple rules feature parameter values that are practically the same as the values retrieved in our estimation: the interest rate smoothing coefficient lies around 0.8 and the coefficient assigned to price inflation lies just above 1. If we interpret the estimated simple rules obtained in Section 3 (which basically coincide for the two models) as arising from optimal policy considerations under uncertainty — where the policymaker intentionally excludes a direct response to wage inflation — U.S. policymakers can be viewed as assigning a probability to the search and matching model being the true data-generating process between 0.6 and 0.8.

5.3.2 Shock persistence and consumption habits

Thus far, we have assumed that the markup shock is transitory and that households do not experience habit persistence in consumption. As our estimation strategy is silent on the parameterization of the markup shock, we also explore the possibility of a mildly persistent markup shock ($\rho_u = 0.2$). Also, the impulse response function matching in Section 3 finds no support for consumption habits in contrast to Christiano, Eichenbaum, and Trabandt (2013). In part, this result emerges as we exclude monetary policy shocks from the empirical analysis. In the SVAR monetary policy shocks induce a pronounced hump-shaped response of consumption and output. One way to capture this feature is to introduce habit persistence in consumption. We investigate the impact of habit persistence in consumption ($h = 0.6$) for our results in a second alternative.

Table 6 summarizes the results for the case of mildly persistent markup shocks. Overall, the results are similar to those in Table 3, if not stronger. Under the Bayesian strategy, the ω-optimal simple rule is biased towards improving the outcomes in the sticky wage model: the welfare loss (measured in CEV) in the sticky wage model is smaller than in the search and matching model as long as $\omega \leq 0.8$ and negligible for $\omega \leq 0.4$ (compared to $\omega \leq 0.2$ in Table 3). The minmax strategy continues to pick the ω-optimal simple rule for $\omega = 0$. Our results also withstand the introduction of habit persistence as shown in Table 7. Yet, in the presence of this real rigidity the bias of the optimal policy under model uncertainty towards the sticky wage model is slightly less pronounced.

6 Conclusion

In this paper, we contrast the optimal monetary policy recommendations in two models that the policymaker views as good approximations of the true data-generating process. The models differ with regard to the details of the labor market. The first model follows the search and matching literature in assuming that workers have to search for jobs and firms post vacancies. For a worker to become employed and for the production of goods to occur, the worker has to be matched with a firm. The two parties then negotiate over the real wage which, as a result, may experience substantial inertia. In the second model, nominal wages are rigid as the result of staggered (nominal) wage contracts. We apply impulse response function matching to estimate key parameters of the models.

While the two models produce very similar impulse responses for common variables under the estimated policy rules, the responses differ importantly when monetary policy is chosen optimally. Under sticky wages without indexation, the optimal policy induces little variation in nominal wages; the dynamics of the real wage are determined by the adjustment in prices.[24] In the search and matching model, it is optimal to stabilize prices and to allow for substantial real wage adjustment brought about by changes in nominal wages. We fill a gap in the literature by deriving the optimal targeting rule for a search and matching model — a single target criterion that seeks to implement the optimal monetary policy. We investigate the performance of each economy under the optimal targeting rule derived in the other model. In particular, the optimal targeting rule derived for the search and matching model is not robust, in the sense that it induces large welfare losses in the sticky wage model. While the optimal targeting rule derived for the sticky wage model also alters the dynamics in the search and matching model relative to the optimal monetary policy in that model, the welfare consequences are less dramatic.

Given the models' sensitivity to the optimal targeting rules, we compute optimal simple (interest rate) rules when the policymaker considers both the sticky wage and the search and matching model to be good candidates for the true data-generating process. Applying Bayesian and minmax strategies to obtain optimal simple rules, we find that unless the policymaker places high probability weight on the search and matching model the optimal simple rule is biased towards stabilizing wage inflation, a feature of the optimal monetary policy in the sticky wage model.

Our framework abstracts from the possibility of learning dynamics as in Svensson and Williams (2005) or Cogley and Sargent (2005). These authors stress how agents adjust their probability distribution over the set of reference models based on observed economic outcomes. In our setup, the implications of learning for policy may be subdued as long as the both models are considered to be sufficiently likely. We plan to explore this issue in future work.

[24] When wages are fully indexed, the optimal policy smoothes the difference between wage and past price inflation.

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Table 1: Calibrated Parameters

Description	Parameter	Search and Matching	Sticky Wage
discount factor	β	0.99	0.99
exogenous seperation rate	ρ	0.1	-
matching function share of unemployment	ζ	0.54	-
steady state inflation rate	$\bar{\pi}$	1	1
Calvo price stickiness	ξ^p	0.75	0.75
steady state price markup	λ^p	1.2	1.2
Calvo wage stickiness	ξ^w	-	0.75
steady state wage markup	λ^w	-	1.2
invers consumption elasticity	σ	1	1
inverse labor supply elasticity	ϕ	2	2
hiring flow cost / output	$100 * \eta_s$	0.66	-
steady state unemployment rate	$\tilde{u}_{ss}$	0.055	-
steady state vacancy filling rate	q_{ss}	0.7	-
steady state working hour	h_{ss}	1/3	1/3
Shock Process			
technology shock: AR	ρ_a	0.9999	0.9999
markup shock: AR	ρ_u	0	0
markup shock: Std	σ_u	0.0104	0.0135
Implied Deep Parameter Value			
hiring fixed cost	$\bar{\kappa}$	0	-
hiring flow cost	κ^v	0.0154	-
unemployment benefit	b^u	0.1769	-
worker's share of surplus	ξ	0.7438	-
matching efficiency	χ	0.6625	-
scaling of working hour disutility	ϕ_0	27.8940	27

Note: Table 1 summarizes the parameters and calibration targets for the NK model with search and matching frictions and the NK model with sticky wages.

Table 2: Estimated Parameters

Description	Estimated Parameter	Search	Sticky Wage	Sticky Wage with Indexation
interest rate smoothing	ρ_R	0.8555	0.8379	0.8895
		[0.0294]	[0.0450]	[0.0260]
weights on inflation	ρ_π	0.1445	0.1622	0.1105
		[1.5e-05]	[3.12e-05]	[2.4e-05]
std technology shock	σ_z	0.0031	0.0033	[0.0031]
		[0.0002]	[0.0002]	[0.0002]
habit persistence	h	0	0	0
		[0.5148]	[0.4394]	[0.7734]
replacement ratio	r^b	0.5345	-	-
		[0.0185]	-	-
price indexation	ι^p	0	0	0
		[0.3123]	[0.3204]	[0.2714]
wage indexation	ι^w	-	-	1
		-	-	[0.1656]

Minimum Distance Estimator

Description	Search	Sticky Wage	Sticky Wage with Indexation
criterion value (9 variables)	124.8128	-	-
criterion value (6 variables)	99.6490	136.0783	77.2143

Note: The top panel of Table 2 summarizes the estimated parameters for the NK model with search and matching frictions and the NK model with and without wage indexation. The parameters are estimated using impulse response function matching under neutral technology shocks. The empirical impulse responses against which the performance of the theoretical models is assessed are taken from the SVAR estimation in Christiano, Eichenbaum, and Trabandt (2013). The numbers in the square bracket are the standard deviations of the estimates. The lower panel provides the value of the criterion function (31) at the minimum.

Table 3: Optimal Simple Rules: transitory markup shock $\rho_u = 0$ and no habit $h = 0$

Approach	Prior	Optimal Simple Rule				Welfare Loss				
		ρ_R	ρ_π	ρ_π^w	ρ_x	Objective	$\mathcal{L}^{s\&m}(\Theta^*)$	$CEV^{s\&m}(\Theta^*)$	$\mathcal{L}^{sw}(\Theta^*)$	$CEV^{sw}(\Theta^*)$
Bayesian	$(0, 1)$	0	0	66.6844	2.3852	3.1047	2.1568	0.1094	3.1047	0.0010
	$(0.1, 0.9)$	0	0	61.5860	2.0019	3.0099	2.1566	0.1092	3.1048	0.0010
	$(0.2, 0.8)$	0	0	56.4038	1.6763	2.9151	2.1565	0.1091	3.1048	0.0011
	$(0.3, 0.7)$	0	0.6240	0.5226	0	2.8139	2.1028	0.0554	3.1186	0.0149
	$(0.4, 0.6)$	0	0.6368	0.5160	0	2.7123	2.1025	0.0551	3.1188	0.0151
	$(0.5, 0.5)$	0	0.6558	0.5131	0	2.6106	2.1022	0.0548	3.1190	0.0153
	$(0.6, 0.4)$	0	0.7005	0.5158	0	2.5088	2.1014	0.0540	3.1199	0.0162
	$(0.7, 0.3)$	0	0.8135	0.5231	0	2.4067	2.0994	0.0520	3.1240	0.0202
	$(0.8, 0.2)$	0	1.1725	0.5245	0	2.3031	2.0920	0.0446	3.1475	0.0438
	$(0.9, 0.1)$	0.8177	0.8860	0	0	2.1870	2.0623	0.0149	3.3098	0.2061
	$(1, 0)$	0.9366	2.1197	0	0	2.0477	2.0477	0.0003	4.0851	0.9814
Minimax		0	0	66.6844	2.3852	3.1047	2.1568	0.1094	3.1047	0.0010
Exchanging OTR							2.1607	0.1133	4.4070	1.3033

Note: Table 3 documents the optimal simple rules when policymakers have two reference models, the NK model with search and matching frictions and the NK model with sticky wages when $\rho_u = 0$ and $h = 0$. For the Bayesian approach, the prior $(\omega, 1-\omega)$ means that probability ω is attached to the search and matching model and probability $(1-\omega)$ is attached to the sticky wage model being the true data-generating process. For the Bayesian and the minmax approach, objective functions are $\mathcal{L}^{Bayesian} = \omega * \mathcal{L}^{s\&m}(\Theta^*) + (1-\omega) * \mathcal{L}^{sw}(\Theta^*)$ and $\mathcal{L}^{minmax} = \max\left\{\mathcal{L}^{s\&m}(\Theta^*), \mathcal{L}^{sw}(\Theta^*)\right\}$ respectively. $\mathcal{L}^{s\&m}(\Theta^*)$ and $CEV^{s\&m}(\Theta^*)$ denote the welfare loss and the consumption equivalent variation for the search and matching model under each optimal simple rule. Similarly, do $\mathcal{L}^{sw}(\Theta^*)$ and $CEV^{sw}(\Theta^*)$ for the sticky wage model. Exchanging OTR refers to exchanging the optimal targeting rules between the two models.

Table 4: Restricted Optimal Simple Rules: $\rho_u = 0$ and $h = 0$

Bayesian Approach	Prior	Restricted Optimal Rule				Welfare Loss		
		ρ_R	ρ_π	ρ_π^w	ρ_x	$\omega * \mathcal{L}^{s\&m}(\Theta^*) + (1-\omega) * \mathcal{L}^{sw}(\Theta^*)$	$\mathcal{L}^{s\&m}(\Theta^*)$	$\mathcal{L}^{sw}(\Theta^*)$
Case I: No interest rate smoothing	$(0, 1)$	0	0	66.6844	2.3852	3.1047	2.1567	3.1047
	$(0.1, 0.9)$	0	0	61.5860	2.0019	3.0099	2.1566	3.1048
	$(0.2, 0.8)$	0	0	56.4038	1.6763	2.9151	2.1565	3.1048
	$(0.3, 0.7)$	0	0.6240	0.5226	0	2.8139	2.1028	3.1186
	$(0.4, 0.6)$	0	0.6368	0.5160	0	2.7123	2.1025	3.1188
	$(0.5, 0.5)$	0	0.6558	0.5131	0	2.6106	2.1022	3.1190
	$(0.6, 0.4)$	0	0.7005	0.5158	0	2.5088	2.1014	3.1199
	$(0.7, 0.3)$	0	0.8135	0.5231	0	2.4067	2.0994	3.1240
	$(0.8, 0.2)$	0	1.1725	0.5245	0	2.3031	2.0920	3.1475
	$(0.9, 0.1)$	0	2.2488	0.4734	0	2.1922	2.0730	3.2651
	$(1, 0)$	0	51.5995	5.2078	0.5170	2.0485	2.0485	20.0584
Case II: No output gap	$(0, 1)$	0	0	100	0	3.1059	2.1556	3.1059
	$(0.1, 0.9)$	0	0	100	0	3.0108	2.1556	3.1059
	$(0.2, 0.8)$	0	0	100	0	2.9158	2.1556	3.1059
	$(0.3, 0.7)$	0	0.6240	0.5226	0	2.8139	2.1028	3.1186
	$(0.4, 0.6)$	0	0.6368	0.5160	0	2.7123	2.1025	3.1188
	$(0.5, 0.5)$	0	0.6558	0.5131	0	2.6106	2.1022	3.1190
	$(0.6, 0.4)$	0	0.7005	0.5158	0	2.5088	2.1014	3.1200
	$(0.7, 0.3)$	0	0.8135	0.5231	0	2.4067	2.0993	3.1242
	$(0.8, 0.2)$	0	1.1725	0.5245	0	2.3031	2.0920	3.1475
	$(0.9, 0.1)$	0.8177	0.8860	0	0	2.1870	2.0623	3.3098
	$(1, 0)$	0.9366	2.1197	0	0	2.0477	2.0477	4.0851
Case III: No price inflation	$(0, 1)$	0	0	66.6844	2.3852	3.1047	2.1567	3.1047
	$(0.1, 0.9)$	0	0	61.5860	2.0019	3.0099	2.1566	3.1048
	$(0.2, 0.8)$	0	0	56.4038	1.6763	2.9151	2.1565	3.1048
	$(0.3, 0.7)$	0	0	93.0510	2.1243	2.8203	2.1563	3.1048
	$(0.4, 0.6)$	0	0	93.5958	2.3938	2.7254	2.1564	3.1048
	$(0.5, 0.5)$	0.5709	0	5.0689	0.2221	2.6309	2.1554	3.1063
	$(0.6, 0.4)$	0.3093	0	20.0193	0.4160	2.5357	2.1559	3.1055
	$(0.7, 0.3)$	0.4090	0	20.0150	0.2744	2.4407	2.1556	3.1060
	$(0.8, 0.2)$	0.5257	0	20.0071	0.1165	2.3457	2.1554	3.1067
	$(0.9, 0.1)$	0.6127	0	19.9914	0	2.2505	2.1553	3.1075
	$(1, 0)$	0.9269	0	19.3246	0	2.1552	2.1552	3.1076
Case IV: No wage inflation	$(0, 1)$	0	1.0001	0	11.2269	3.1107	2.2809	3.1107
	$(0.1, 0.9)$	0.0473	0.9528	0	2.3993	3.0267	2.2599	3.1119
	$(0.2, 0.8)$	0.9900	0.0113	0	0.0011	2.9370	2.1308	3.1386
	$(0.3, 0.7)$	0.9900	0.0112	0	7.5300e04	2.8357	2.1232	3.1410
	$(0.4, 0.6)$	0.9900	0.0111	0	5.1800e04	2.7335	2.1178	3.1439
	$(0.5, 0.5)$	0.9142	0.1019	0	5.5770e04	2.6296	2.1052	3.1539
	$(0.6, 0.4)$	0.9079	0.1171	0	0	2.5243	2.1024	3.1572
	$(0.7, 0.3)$	0.8947	0.2260	0	0	2.4176	2.0912	3.1790
	$(0.8, 0.2)$	0.8669	0.4487	0	0	2.3061	2.0769	3.2226
	$(0.9, 0.1)$	0.8177	0.8861	0	0	2.1870	2.0623	3.3099
	$(1, 0)$	0.9367	2.1182	0	0	2.0477	2.0477	4.0846

Note: Table 4 reports the optimal simple rules under the Bayesian approach similar to Table 3 when restricting the rule not to respond to one of the variables in (48) at the time.

Table 5: Euclidean Distance of IRFs under robust Taylor rules

	Search Model		Stickywage Model	
	Technology Shock	Price Markup Shock	Technology Shock	Price Markup Shock
Reference Prior	(1,0)	(1,0)	(0,1)	(0,1)
Prior (0,1)	0.0042	0.0522	0	0
Prior (0.1,0.9)	0.0042	0.0522	0.0000	0.0001
Prior (0.2,0.8)	0.0042	0.0522	0.0001	0.0003
Prior (0.3,0.7)	0.0033	0.0407	0.0003	0.0077
Prior (0.4,0.6)	0.0032	0.0405	0.0003	0.0077
Prior (0.5,0.5)	0.0032	0.0402	0.0004	0.0078
Prior (0.6,0.4)	0.0031	0.0397	0.0006	0.0080
Prior (0.7,0.3)	0.0030	0.0386	0.0011	0.0086
Prior (0.8,0.2)	0.0027	0.0347	0.0024	0.0116
Prior (0.9,0.1)	0.0001	0.0191	0.0098	0.0245
Prior (1,0)	0	0	0.0157	0.0659

Note: Table 5 documents the Euclidean distance between the impulse response functions (IRFs) for price inflation, wage inflation, and output under the ω-optimal simple rule for both models. Reference prior (1,0) means that the IRFs of the search and matching model are taken as reference to compute the Euclidean distance, whereas reference prior (0,1) means that IRFs of sticky wage model are taken as reference.

Table 6: Optimal Simple Rules: persistent markup shock $\rho_u = 0.2$ and no habit $h = 0$

Approach	Prior	Optimal Simple Rule				Welfare Loss				
		ρ_R	ρ_π	ρ_π^w	ρ_x	Objective	$\mathcal{L}^{s\&m}(\Theta^*)$	$CEV^{s\&m}(\Theta^*)$	$\mathcal{L}^{sw}(\Theta^*)$	$CEV^{sw}(\Theta^*)$
Bayesian	$(0, 1)$	0	0	66.6812	1.6100	3.2524	2.2305	0.1520	3.2524	0.0014
	$(0.1, 0.9)$	0	0	62.1310	1.3100	3.1501	2.2302	0.1518	3.2524	0.0014
	$(0.2, 0.8)$	0	0	57.0042	1.0663	3.0479	2.2300	0.1516	3.2524	0.0015
	$(0.3, 0.7)$	0	0	51.9820	0.8663	2.9457	2.2299	0.1514	3.2524	0.0015
	$(0.4, 0.6)$	0	0	47.4445	0.6725	2.8434	2.2297	0.1513	3.2525	0.0016
	$(0.5, 0.5)$	0	0.6436	0.5084	0	2.7087	2.1448	0.0663	3.2726	0.0217
	$(0.6, 0.4)$	0	0.6743	0.5110	0	2.5958	2.1441	0.0656	3.2735	0.0226
	$(0.7, 0.3)$	0	0.7419	0.5165	0	2.4827	2.1424	0.0639	3.2768	0.0259
	$(0.8, 0.2)$	0	0.9590	0.5250	0	2.3683	2.1361	0.0576	3.2973	0.0463
	$(0.9, 0.1)$	0.8744	0.5682	0	0	2.2419	2.0993	0.0208	3.5253	0.2744
	$(1, 0)$	0.9789	1.4644	0	0	2.0785	2.0785	0.0000	4.7228	1.4719
Minimax		0	0	66.6812	1.6100	3.2524	2.2305	0.1520	3.2524	0.0014
Exchanging OTR							2.2372	0.1588	5.5388	2.2879

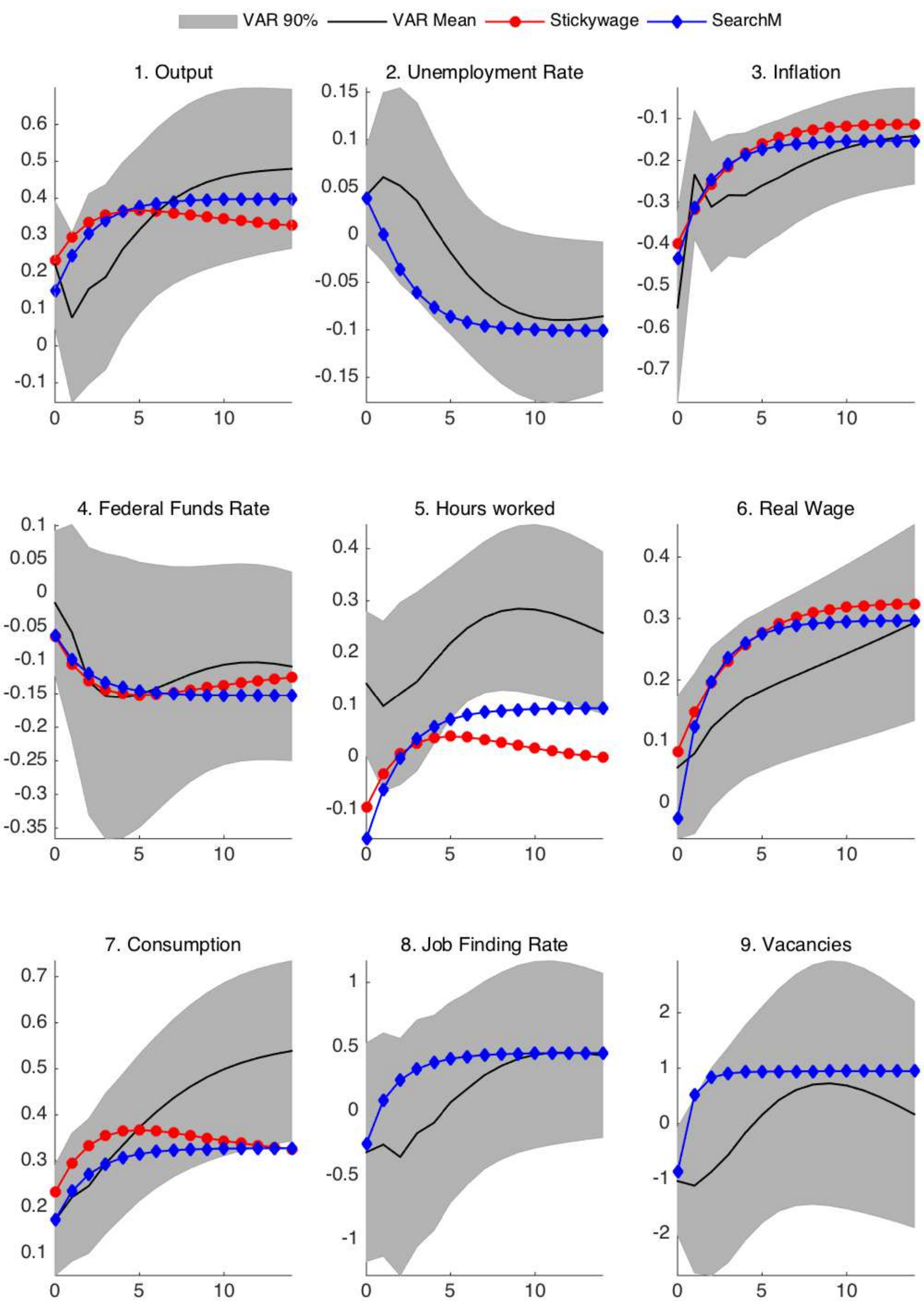
Note: Table 6 reports the optimal simple rules under the Bayesian and minmax approach similar to Table 3 when the two models feature persistent markup shocks $\rho_u = 0.2$ and no habit persistence $h = 0$.

Table 7: Optimal Simple Rules: transitory markup shock $\rho_u = 0$ and habit $h = 0.6$

Approach	Prior	Optimal Simple Rule				Welfare Loss				
		ρ_R	ρ_π	ρ_π^w	ρ_x	Objective	$\mathcal{L}^{s\&m}(\Theta^*)$	$CEV^{s\&m}(\Theta^*)$	$\mathcal{L}^{sw}(\Theta^*)$	$CEV^{sw}(\Theta^*)$
Bayesian	$(0, 1)$	0	0	68.1424	1.4345	3.1519	2.1796	0.1347	3.1519	0.0007
	$(0.1, 0.9)$	0	0.6175	1.3949	0	3.0546	2.1689	0.1240	3.1530	0.0018
	$(0.2, 0.8)$	0	0.6561	1.3497	0	2.9561	2.1678	0.1229	3.1532	0.0020
	$(0.3, 0.7)$	0	0.7064	1.3276	0	2.8575	2.1664	0.1215	3.1537	0.0025
	$(0.4, 0.6)$	0	0.7754	1.3139	0	2.7586	2.1646	0.1197	3.1547	0.0035
	$(0.5, 0.5)$	0	0.8786	1.3099	0	2.6594	2.1621	0.1172	3.1568	0.0056
	$(0.6, 0.4)$	0.9882	1.4557	0	0.2369	2.5576	2.1205	0.0756	3.2132	0.0620
	$(0.7, 0.3)$	0.9198	2.4368	0	0.3228	2.4461	2.1042	0.0593	3.2438	0.0926
	$(0.8, 0.2)$	0.8178	4.0965	0	0.4114	2.3289	2.0867	0.0418	3.2976	0.1464
	$(0.9, 0.1)$	0.6763	6.9621	0	0.4346	2.2020	2.0677	0.0228	3.4110	0.2598
	$(1, 0)$	0	41.4453	1.6503	0	2.0465	2.0465	0.0016	5.0518	1.9006
Minimax		0	0	68.1424	1.4345	3.1519	2.1796	0.1347	3.1519	0.0007

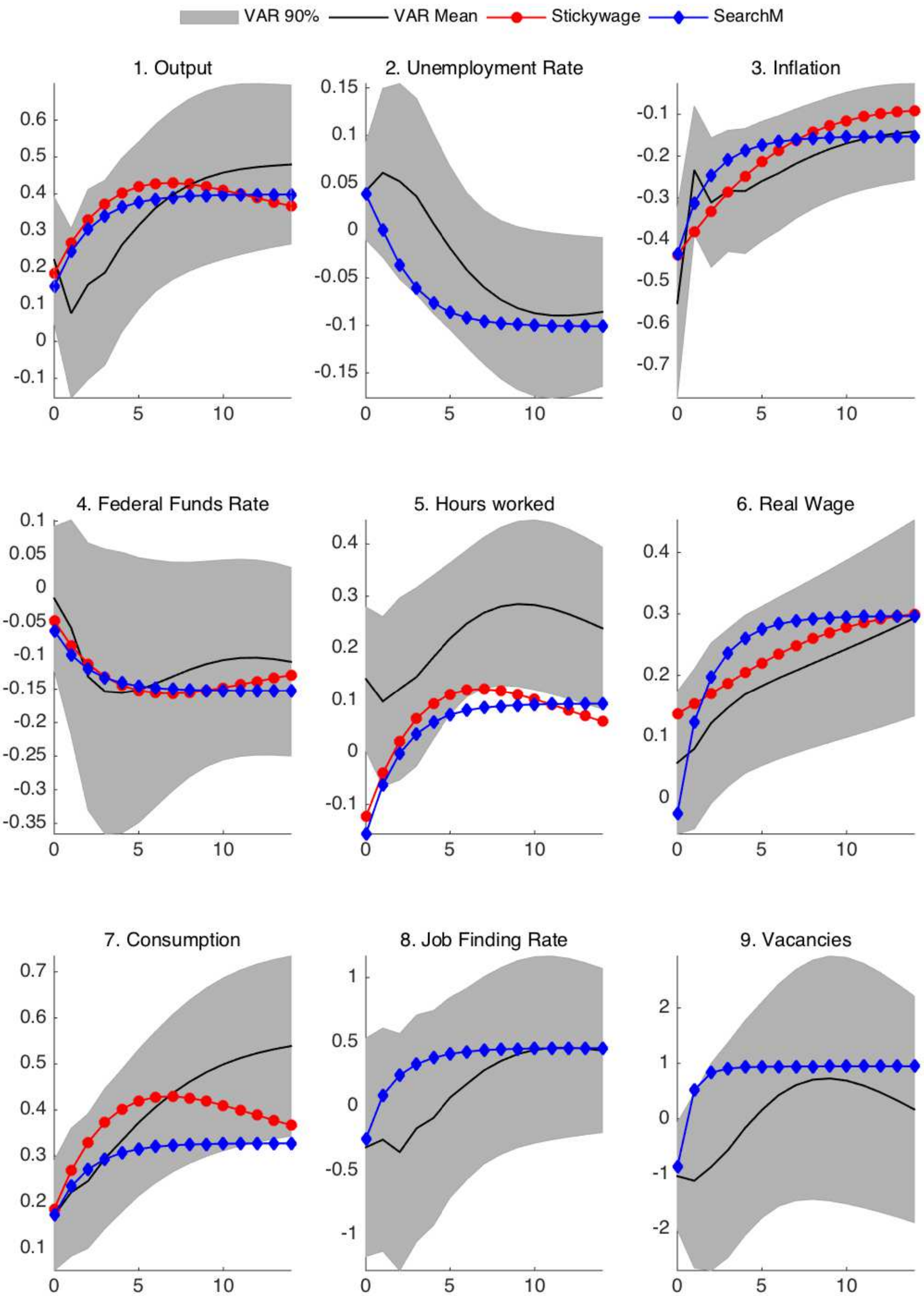
Note: Table 7 reports the optimal simple rules under the Bayesian and minmax approach similar to Table 3 when the two models feature transitory markup shocks $\rho_u = 0$ and habit persistence $h = 0.6$.

Figure 1: Impulse response function matching under neutral technology shock



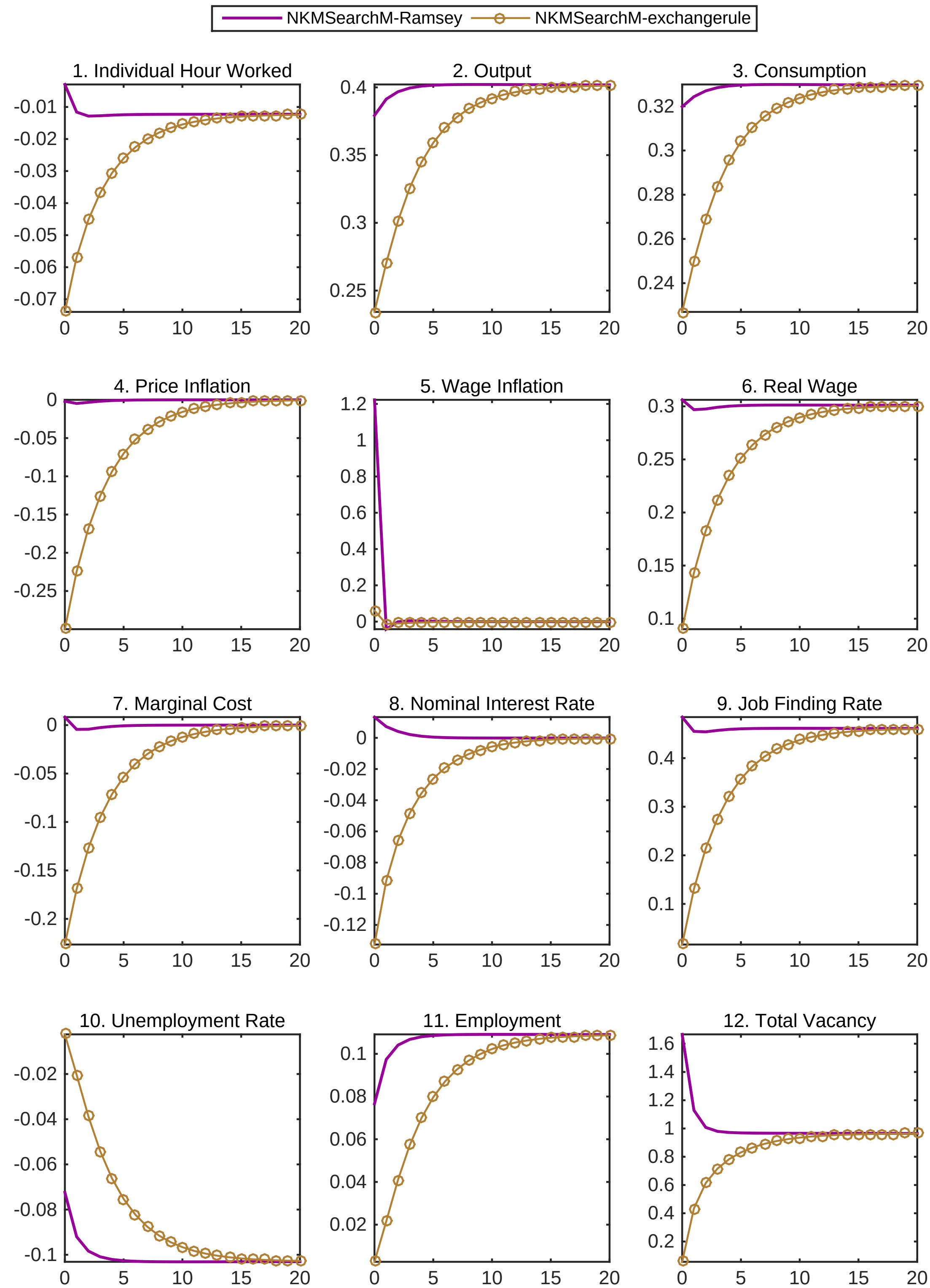
Note: Figure 1 depicts the impulse responses to a neutral technology shock in the search and matching model (blue) and the sticky wage model (red). The solid black lines show the point estimates of the empirical impulse responses along with the 90% confidence interval, the grey shaded area. Inflation rates and the federal fund rate are annualized.

Figure 2: Impulse response function matching under neutral technology shock with wage indexation in the sticky wage model



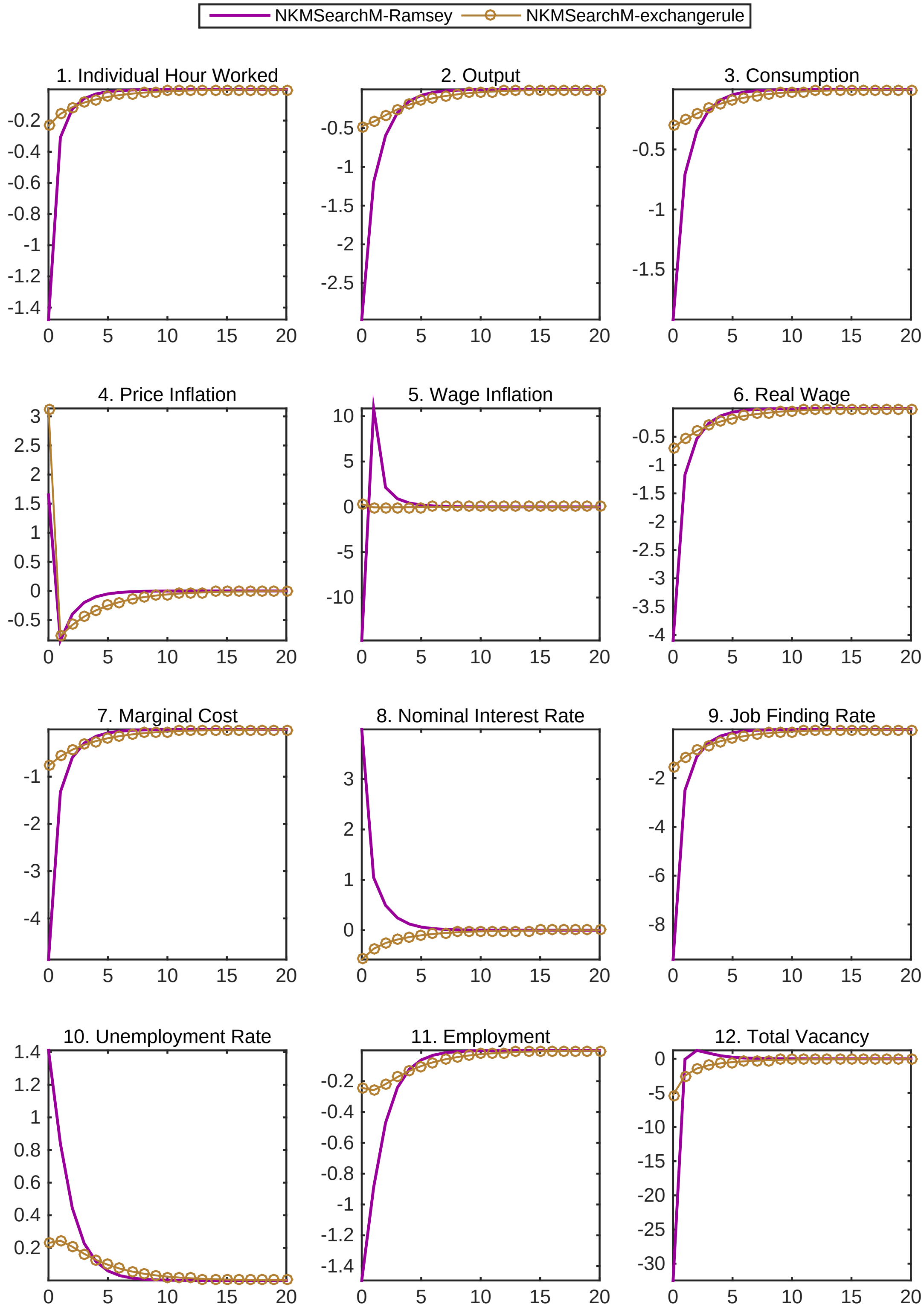
Note: Figure 2 depicts the impulse responses to a neutral technology shock in the search and matching model (blue) and the sticky wage model (red). The solid black lines show the point estimates of the empirical impulse responses along with the 90% confidence interval, the grey shaded area. Inflation rates and the federal fund rate are annualized.

Figure 3: Targeting rules in the search and matching model: neutral technology shock



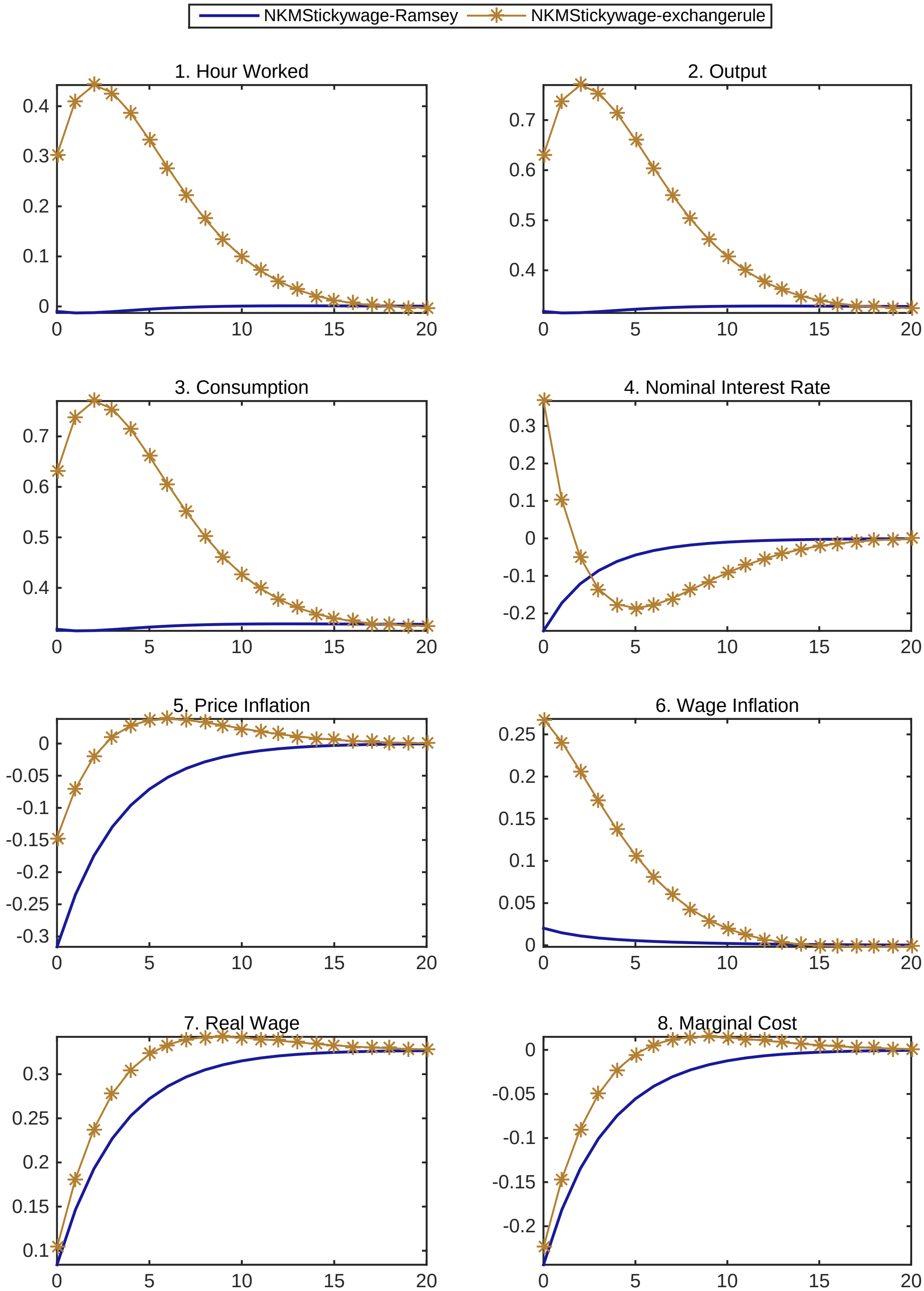
Note: Figure 3 plots the impulse responses in the search and matching model to a neutral technology shock when policy follows the optimal targeting rule from the search and matching model (purple) and the sticky wage model (yellow).

Figure 4: Targeting rules in the search and matching model: price markup shock



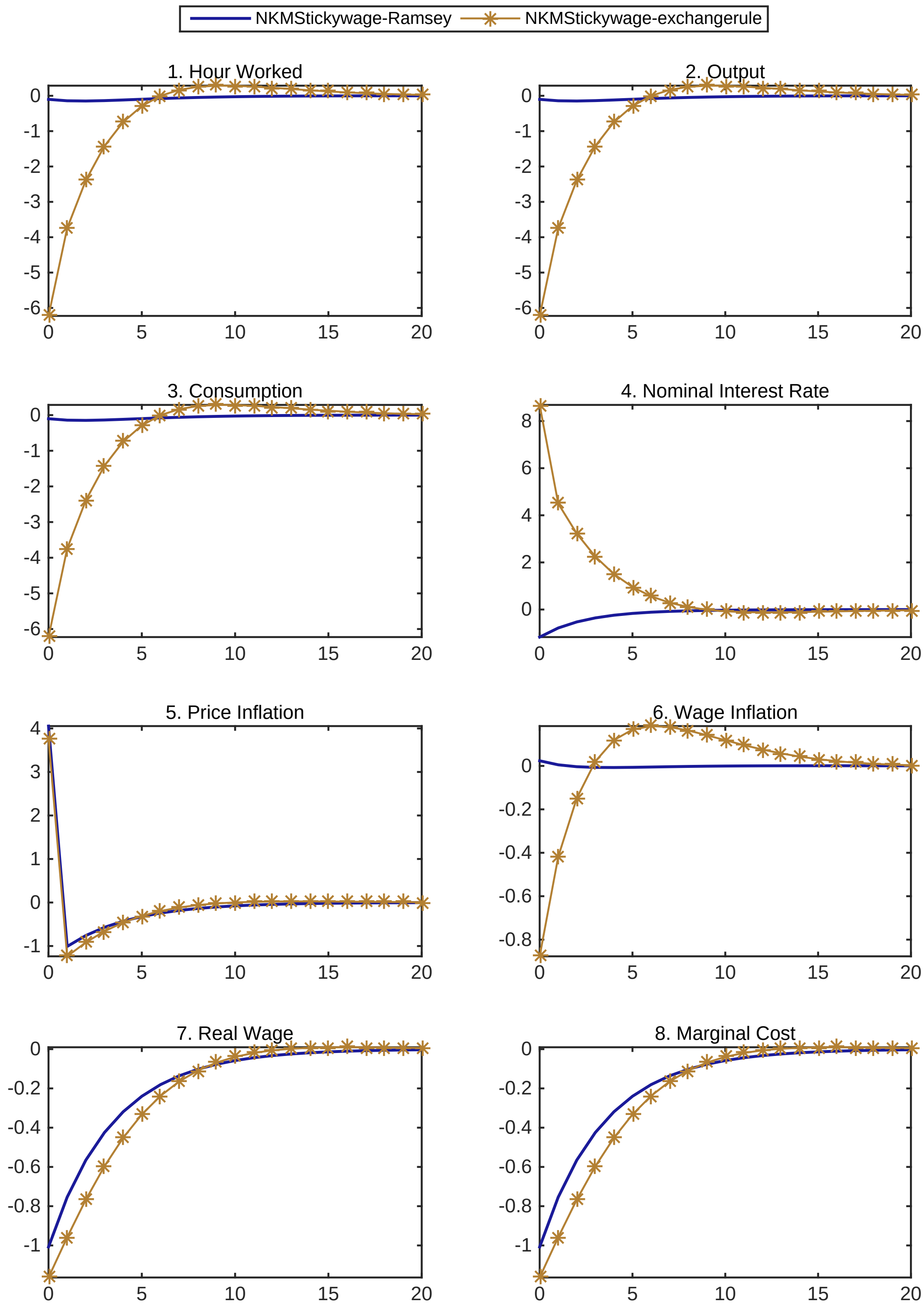
Note: Figure 4 plots the impulse responses in the search and matching model to a price markup shock when policy follows the optimal targeting rule from the search and matching model (purple) and the sticky wage model (yellow).

Figure 5: Targeting rules in the sticky wage model: neutral technology shock



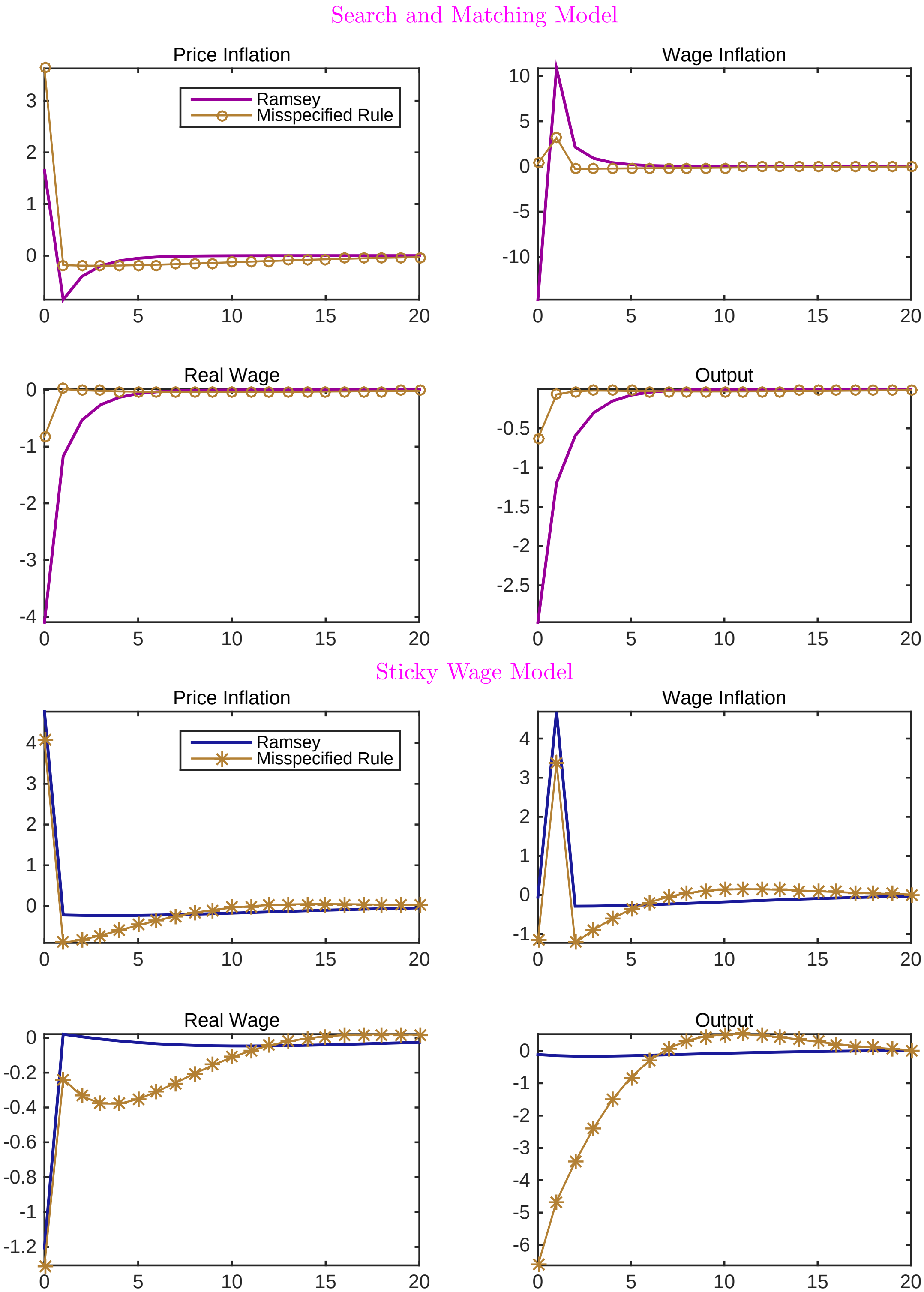
Note: Figure 5 plots the impulse responses in the sticky wage model to a neutral technology shock when policy follows the optimal targeting rule from the sticky wage model (blue) and the search and matching model (yellow).

Figure 6: Targeting rules in the sticky wage model: price markup shock



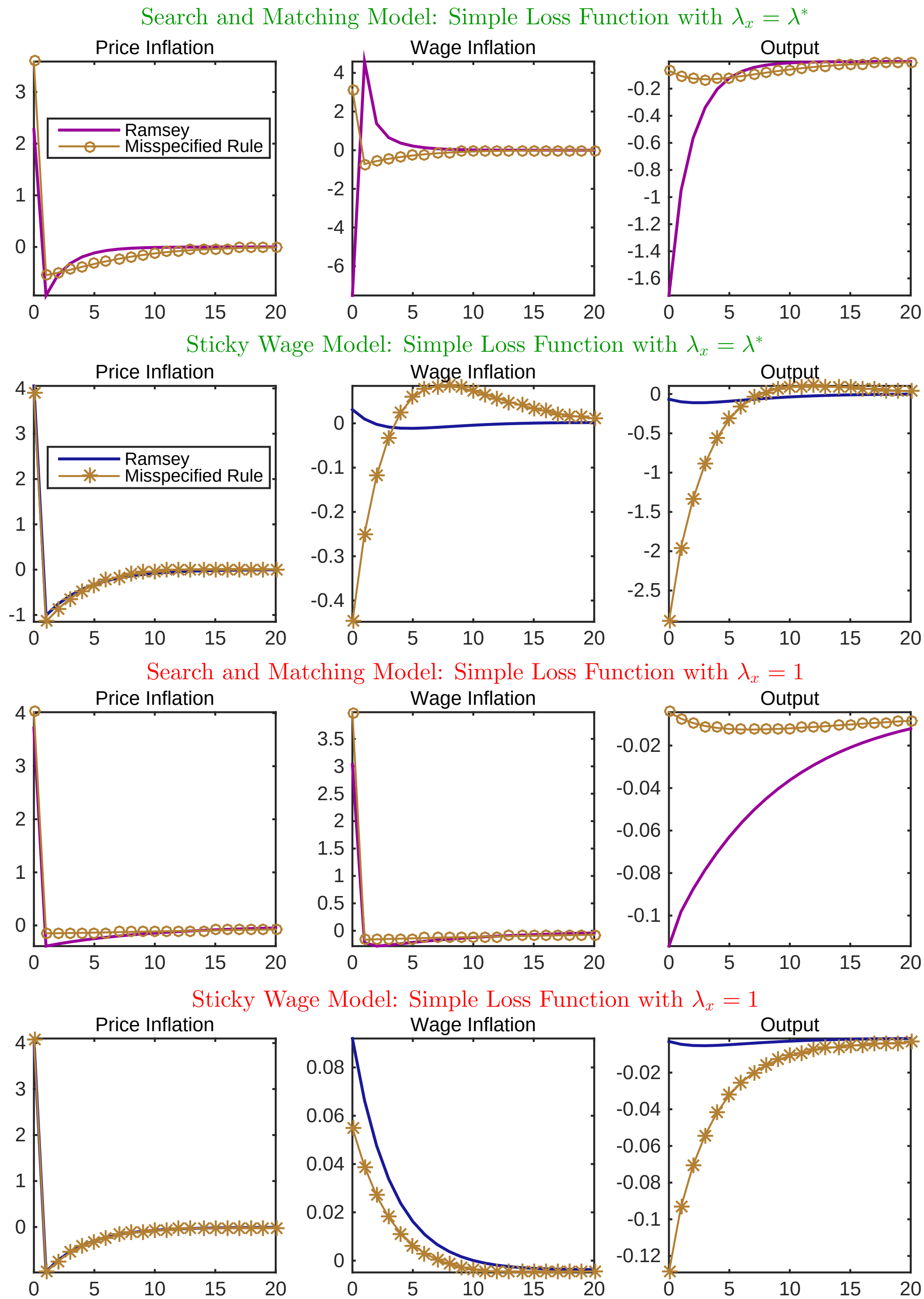
Note: Figure 6 plots the impulse responses in the sticky wage model to a price markup shock when policy follows the optimal targeting rule from the sticky wage model (blue) and the search and matching model (yellow).

Figure 7: Targeting rules with wage indexation in the sticky wage model: price markup shock



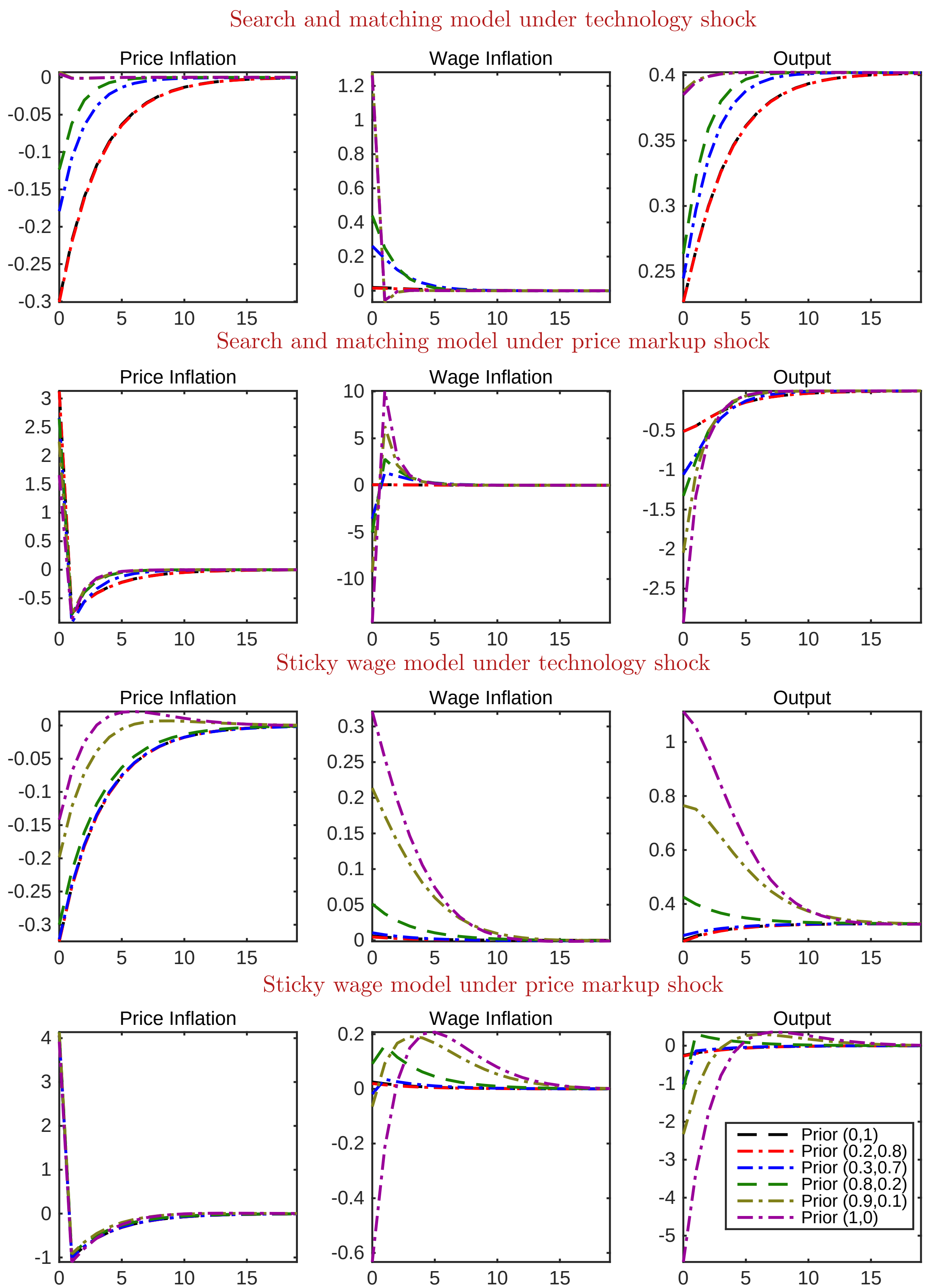
Note: Figure 7 compares the performance of optimal targeting rules for both the search and matching model and the sticky wage model in response to a *price markup shock* when the sticky wage model features wage indexation.

Figure 8: Targeting rules with simple loss function: price markup shock



Note: Figure 8 compares the performance of optimal targeting rules derived from the loss function $\left(\pi_t^2 + \lambda_x x_t^2\right)$ for both the search and matching model and the sticky wage model in response to a *price markup shock*. In the upper six panels, it is $\lambda_x = \lambda^* = 0.0429$; in the lower six panels it is $\lambda_x = 1$.

Figure 9: Impulse responses under optimal simple rules



Note: Figure 9 compares the performance of the search and matching and the sticky wage model under ω-optimal simple rules $(0,1)$, $(0.2,0.8)$, $(0.3,0.7)$, $(0.8,0.2)$, $(0.9,0.1)$, and $(1,0)$ for the neutral technology shock and the price markup shock.

Appendix to: "Employment, Wages and Optimal Monetary Policy"

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December 2016

A Two reference models with labor market frictions

This Appendix provides more details on the two models laid out in the main text.

A.1 NK model with search and matching frictions

A.1.1 Household

Each household consists a continuum of agents with measure 1. n_t agents of the household are employed and $(1 - n_t)$ agents are unemployed. The Household maximizes the weighted utility of employed and unemployed agents. The inter-temporal preferences of the household are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[n_t U(c_t^w, h_t^w) + (1 - n_t) U(c_t^u, h_t^u) \right]. \tag{49}$$

$\mathbb{E}_0$ is the expectations operator conditional on all the information available up to period 0. β is time discount factor. The variable c_t^w stands for consumption of employed agents at time t, and c_t^u represents consumption of unemployed agents at time t. Similarly, h_t^w is the number of hours worked by employed agents, and h_t^u is the number of hours worked by unemployed agents.

Employed agents derive utility from consumption c_t^w, but also incur disutility from hours worked h_t^w

$$U(c_t^w, h_t^w) = \frac{(c_t^w - hc_{t-1}^w)^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} ({h_t}^w)^{1+\phi}. \tag{50}$$

σ captures the intertemporal elasticity of consumption, h measures the degree of habit persistence, and ϕ is the inverse Frisch labor supply elasticity. Unemployed agents get utility from consumption c_t^u only; their labor input is normalized to zero

$$U(c_t^u, h_t^u) = \frac{(c_t^u - hc_{t-1}^u)^{1-\sigma}}{1-\sigma}. \tag{51}$$

Total consumption for the household is defined as

$$c_t = n_t c_t^w + (1 - n_t) c_t^u. \tag{52}$$

The inter-temporal budget constraint of the household satisfies

$$n_t c_t^w + (1 - n_t) c_t^u + \frac{B_{t+1}}{P_t} \leqslant w_t h_t^w n_t + b^u (1 - n_t) + \frac{Pr_t}{P_t} + \frac{T_t}{P_t} + \frac{R_{t-1} B_t}{P_t}. \tag{53}$$

The household earns labor income $W_t n_t h_t^w$ from labor services supplied by employed agents, and $b^u(1-n_t)$ in terms of unemployment benefits (generated through home production) from the unemployed agents. Furthermore, the household receives payments from last period's bond holdings $R_{t-1}B_t$, government transfers T_t, and an aliquot share of profits Pr_t. P_t is the price of the final consumption good, and R_t denotes the gross return on the one period risk free bond B_t. Each period t, the household chooses consumption, bond holdings to maximise the expected discounted value of lifetime utility, taking prices and transfers as given. The household is not choosing the level of total employment, wages, or hours worked. Hours and wages are set in a bargaining game between the individual worker and the firm, to which the worker is matched, over the surplus of the match. However, the marginal value of employment to the household is a key component in determining the surplus of the match, see below.

The first order conditions with respect to c_t^w and c_t^u deliver

$$\lambda_t = U_c(c_t^w, h_t^w) \tag{54}$$

$$\lambda_t = U_c(c_t^u, h_t^u) \tag{55}$$

where λ_t denotes the Lagrange multiplier associated with equation (53), household's budget constraint. Equations (54) and (55) imply that consumption risk is fully shared

$$c_t^w = c_t^u = c_t. \tag{56}$$

Hence, the household's problem can be written more compactly as with preferences

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{(c_t - hc_{t-1})^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} h_t^{1+\phi} n_t \right] \tag{57}$$

and the budget constraint

$$c_t + \frac{B_{t+1}}{P_t} = \frac{W_t}{P_t} n_t h_t + b^u(1-n_t) + \frac{R_{t-1}B_t}{P_t} + \frac{Pr_t}{P_t} + \frac{T_t}{P_t}. \tag{58}$$

A.1.2 Employment and the labor market

The labor market is characterized by search and matching frictions. In this economy, the presence of search and matching frictions impedes people who are seeking jobs from finding one and wholesale firms that are posting vacancies from filling them. At the beginning of each period, a share ρ of matches that existed in the last period n_{t-1} break up. The share $(1-\rho)$ of matches survives. With the labor force normalised to unity, the total number of job seekers in period t is the sum of unemployed workers in period $t-1$ and the newly fired workers. Let u_t denote the total number of job seekers,

$$u_t = 1 - n_{t-1} + \rho n_{t-1} = 1 - (1-\rho)\, n_{t-1} \tag{59}$$

The unemployment rate differs from u_t as some job seekers may be matched and become employed. We define the unemployment rate

$$\tilde{u}_t = 1 - n_t. \tag{60}$$

Firms post vacancies v_t to be filled with job-seeking workers. Unemployed workers are matched to vacant jobs according to the constant returns to scale matching function

$$m_t = \chi u_t^{\zeta} v_t^{1-\zeta}. \tag{61}$$

χ determines the degree of matching efficiency, ζ captures the curvature of Beveridge curve, indicating the substitutability between vacancies and job seekers. Newly formed matches m_t result immediately in employment. The latter evolves according to

$$n_t = (1-\rho)\, n_{t-1} + m_t. \tag{62}$$

Finally, we define the job finding rate s_t as the probability of an unemployed worker being matched to a vacant job

$$s_t = \frac{m_t}{u_t} = \chi \theta^{1-\zeta}. \tag{63}$$

The vacancy filling rate q_t is the probability for a vacancy being filled

$$q_t = \frac{m_t}{v_t} = \chi \theta^{-\zeta}. \tag{64}$$

Labor market tightness θ_t is defined as

$$\theta_t = \frac{v_t}{u_t}. \tag{65}$$

We are now in a position to define the marginal value of employment to the household H_t consistent with the household's optimization problem

$$H_t = \frac{W_t}{P_t} h_t - b^u - \frac{\phi_0}{1+\phi} h_t^{1+\phi} \frac{1}{\lambda_t} + (1-\rho)\, E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1 - s_{t+1})\, H_{t+1}. \tag{66}$$

Moving one household member into employment affects the household in three ways.

First, total household resources rise by the difference between wages and unemployment benefits. Second, the utility of the agent changing employment status falls by the disutility from labor (divided by the marginal utility of wealth λ_t to turn it into monetary terms). Finally, the gains from matching a household member with a firm also occur in future periods.

A.1.3 Wholesale firms

Wholesale firms employ labor as the only factor of production. Their output is sold at the competitive market price P_t^w. Firms post vacancies at the flow vacancy posting cost κ^v. A wholesale firm's optimization problem is

$$\begin{aligned} & \max_{\{y_t, v_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left(\frac{P_t^w}{P_t} y_t^w - \frac{W_t}{P_t} n_t h_t - \kappa^v v_t \right) \\ & s.t. \quad n_t = (1-\rho)\, n_{t-1} + q_t v_t \\ & \qquad\quad y_t^w = a_t n_t h_t. \end{aligned} \tag{67}$$

The technology shock a_t follows an exogenous AR(1) process

$$\log(a_t) = \rho_a \log(a_t) + \varepsilon_t^a \tag{68}$$

with ε_t^a given by $N(0, \sigma_a^2)$.

Let J_t denote the marginal value of employment to the wholesale firm (the lagrange multiplier associated with the first constraint). The first order condition with respect to vacancy postings implies

$$q_t J_t = \kappa^v. \tag{69}$$

Using the envelop theorem J_t itself is defined as

$$J_t = \left(\frac{P_t^w}{P_t} a_t h_t - \frac{W_t}{P_t} h_t \right) + (1-\rho)\, E_t \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}. \tag{70}$$

Employing one additional worker raises the firm's profits in the current period by the increment between marginal product of labor and wage payment. Furthermore, if the match survives future the firm also enjoys a continuation value.

Combining equations, the wholesale firm's vacancy posting condition equation (70) can be rewritten as

$$mc_t a_t h_t = \frac{W_t}{P_t} h_t + \frac{\kappa^v}{q_t} - (1-\rho) E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa^v}{q_{t+1}}. \tag{71}$$

The wholesale firms' real revenue $\frac{P_t^w}{P_t}$ is in effect the intermediate firms' real marginal cost mc_t. The left hand side of equation (71) indicates the marginal benefit of hiring an additional worker. The right hand side of equation (71) captures the marginal cost of hiring a new worker, involving wage payments for hours worked, vacancy posting costs associated with a new match, and the present value of saved future vacancy posting costs if the match survives in following periods.

Notice that the search and matching frictions work through the presence of vacancy posting costs. Absent vacancy posting costs, wholesale firms would post infinitely many vacancies. All the unemployed workers seeking jobs will find one. The NK model with search and matching frictions reduces to the standard NK model and marginal costs would be given by $mc_t = \frac{w_t}{a_t}$.

A.1.4 Wage bargaining

The real wage w_t and hours worked are determined by Nash bargaining between the worker and the firm after forming a match. The total surplus of the match is given by

$$J_t + H_t. \tag{72}$$

Under Nash bargaining the solution to the bargaining game is obtained from

$$\max_{w_t, h_t} J_t^{1-\xi} H_t^{\xi} \tag{73}$$

subject to equations (70) and (66). ξ stands for the bargaining power of the household, and $1-\xi$ indicates the bargaining power of the firm.

The sharing rule for this Nash bargaining mechanism as derived from the first order condition with respect to w_t implies

$$\xi J_t = (1-\xi) H_t. \tag{74}$$

Combining equations (70), (66) and (74) yields an expression for the bargained wage

$$w_t h_t = \xi \left(h_t mc_t a_t + (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} s_{t+1} J_{t+1} \right] \right) + (1-\xi) \left(b^u + \frac{\phi_0}{1+\phi} h_t^{1+\phi} \frac{1}{\lambda_t} \right). \tag{75}$$

Combining equation (71) and equation (75), we obtain the equilibrium condition for vacancy posting

$$\frac{\kappa^v}{q_t} = (1-\xi)\left(h_t mc_t a_t - b^u - \frac{\phi_0}{1+\phi} h_t^{1+\phi} \frac{1}{\lambda_t}\right) + (1-\rho)\, E_t \beta \frac{\lambda_{t+1}}{\lambda_t} (1-\xi s_{t+1}) \frac{\kappa^v}{q_{t+1}}. \quad (76)$$

The first order condition associated with hours worked in the Nash bargaining problem can be written as

$$mc_t a_t = \frac{\phi_0 {h_t}^{\phi}}{\lambda_t}. \quad (77)$$

A.1.5 Retailers

Retail good producers purchase wholesale goods to produce differentiated intermediate good varieties. The retailers have monopoly power over their variety. The retailer's cost minimization problem is then given by

$$\begin{aligned} &\min_{y_t^w(i), Y_t(i)} \; P_t^w y_t^w(i) \\ &s.t. \quad Y_t(i) = y_t^w(i) \end{aligned} \quad (78)$$

with the first order condition for $y_t^w(i)$ being

$$P_t^w - \lambda_t^w = 0 \quad (79)$$

where λ_t^w is the Lagrange multiplier for the production function and thus represents the marginal cost. Therefore, real marginal costs satisfy

$$\frac{P_t^w}{P_t} = mc_t. \quad (80)$$

The prices of intermediate goods $P_t(i)$ are determined by Calvo-style staggered contracts, see Calvo (1983). Each period, a firm faces a constant probability $1-\xi^p$ to re-optimize its price $P_t(i)$. The probability is independent across firms and across time. For those firms that do not re-optimize their price, prices will be updated as a weighted average of $\Pi_t = \frac{P_t}{P_{t-1}}$ the nominal price inflation in the last period and $\bar{\Pi}$ the steady state inflation rate. The relative importance of Π_t and $\bar{\Pi}$ is governed by price indexation parameter ι^p.[25] More specifically,

$$P_{t+1}(i) = \tilde{P}_t(i)\left({\pi_t}^{\iota_p} \bar{\pi}^{1-\iota_p}\right). \quad (81)$$

[25] This price updating scheme avoids price dispersions in steady state if the steady state inflation rate is not zero.

Price setting behavior of intermediate good firm i is derived from

$$\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi^p \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left[\left((1+\bar{\tau}^p) \tilde{P}_t(i) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^p} \bar{\Pi}^{1-\iota^p} \right) - MC_{t+s} \right) \right] Y_{t+s}(i)$$

$$s.t. \quad Y_{t+s}(i) = \left(\frac{\tilde{P}_t(i) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^p} \bar{\Pi}^{1-\iota^p} \right)}{P_{t+s}} \right)^{-\dfrac{\lambda^p}{\lambda^p - 1}} Y_{t+s}. \tag{82}$$

$\bar{\tau}^p$ is the subsidy to intermediate firms. We assume $\bar{\tau}^p = \lambda^p - 1$ to remove the distortions arising from monopolistic competition between the retailers. We introduce markup shocks in the first order conditions for intermediate firms. We define $\theta^p = \lambda^p - 1$.

A.1.6 Final good producer

Differentiated intermediate products are combined to form the composite goods by a continuum of representative bundlers in a perfectly competitive environment based on the CES aggregator

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\lambda^p}} \mathrm{d}i \right]^{\lambda^p} \tag{83}$$

where $\frac{\lambda^p}{\lambda^p-1}$ refers to the elasticity of substitution between intermediate varieties. Profit maximisation of a bundler is defined as

$$\max_{Y_t(i), Y_t} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

$$s.t. \qquad Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\lambda^p}} \mathrm{d}i \right]^{\lambda^p}. \tag{84}$$

The first order conditions can be recombined to obtain the demand function for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\lambda^p}{\lambda^p-1}} Y_t \tag{85}$$

and the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda^p-1}} \mathrm{d}i \right]^{-(\lambda^p-1)}. \tag{86}$$

A.2 NK model with Calvo sticky wage

We only describe the parts of the model that are different from the search and matching model. More details are provided in Erceg, Henderson, and Levin (2000).

A.2.1 Household

Each Household maximizes preferences

$$E_0 \sum_{t=0}^{\infty} \beta^{t-t_0} \left[\frac{\left(c_t(j) - hc_{t-1}(j)\right)^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} h_t(j)^{1+\phi} \right] \tag{87}$$

subject to the budget constraint

$$P_t c_t(j) + B_{t+1}(j) = (1 + \bar{\tau}^w)\, W_t(j) h_t(j) + R_{t-1} B_t(j) + Pr_t(j) + T_t(j). \tag{88}$$

$\mathbb{E}_0$ is the expectations operator conditional on all the information available up to period 0. β is the time discount factor. The variable $c_t(j)$ stands for household consumption. h indicates the degree of internal consumption habits. P_t is the price of consumption goods, and R_t denotes the gross return for the one period risk free bond $B_t(j)$. The Household earns income by supplying labor services $W_t(j)h_t(j)$, receives payments from last period bond holding $R_{t-1}B_t(j)$, and $Pr_t(j)$ which consists of an aliquot share of profits distributed. Finally, the household receives the government transfer $T_t(j)$. ϕ represents the inverse Frisch labor supply elasticity. Labor income $W_t(j)h_t(j)$ is subsidized at a fix rate $\bar{\tau}^w$.

A.2.2 Labor bundler

Labor bundlers package differentiated labor services supplied by each individual into an aggregate labor service with a CES technology resold to the intermediate good producers in perfectly competitive markets. The labor bundling technology is specified as

$$h_t = \left[\int_0^1 h_t(j)^{\frac{1}{\lambda^w}} \mathrm{d}j \right]^{\lambda^w} \tag{89}$$

where $\frac{\lambda^w}{\lambda^w - 1}$ refers to the elasticity of substitution between differentiated labor types. We define $\theta^w = \lambda^w - 1$.

Labor bundlers maximize profits in a perfectly competitive environment. Profit maximization for labor bundlers implies

$$\max_{h_t(j),h_t} W_t h_t - \int_0^1 W_t(j) h_t(j) dj$$
$$s.t. \quad h_t = \left[\int_0^1 h_t(j)^{\frac{1}{\lambda^w}} \mathrm{d}j\right]^{\lambda^w}.$$

The first order conditions imply that the demand for differentiated labor services satisfies

$$h_t(j) = \left[\frac{W_t(j)}{W_t}\right]^{-\frac{\lambda^w}{\lambda^w-1}} h_t \tag{90}$$

with the aggregate (nominal) wage being defined as

$$W_t = \left[\int_0^1 W_t(j)^{-\frac{1}{\lambda^w-1}} \mathrm{d}j\right]^{-(\lambda^w-1)}. \tag{91}$$

A.2.3 Wage setting

Households supply their differentiated labor services to the labor bundlers. There is a continuum of households, index by $j \in (0, 1)$. The imperfect substitutability of differentiated labor gives each individual household certain degree of market power in setting a nominal wage. Each monopolistic household chooses labor supply $h_t(j)$ and wage setting $W_t(j)$. In addition, wage setting is subject to nominal rigidities as in Calvo (1983). As in Erceg, Henderson, and Levin (2000), households can readjust nominal wages with probability $1-\xi^w$ in each period. For those that cannot adjust wages, wages will increase by the weighted average of inflation in the last period Π_t and the steady state inflation rate $\bar{\Pi}$

$$W_{t+1}(j) = \tilde{W}_t(j)\left(\Pi_t^{\iota^\omega}\bar{\Pi}^{1-\iota^\omega}\right). \tag{92}$$

For those that can re-optimize, the problem is to choose a wage $\tilde{W}_t(j)$ that maximizes its utility in all states of nature where the household has to maintain that wage in the future

$$\max_{\tilde{W}_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\xi^w \beta)^s \left[\frac{(c_{t+s} - hc_{t-1+s})^{1-\sigma}}{1-\sigma} - \frac{\phi_0}{1+\phi} h_{t+s}(j)^{1+\phi}\right]$$
$$s.t. \quad P_{t+s}c_{t+s} + B_{t+s+1} = (1+\bar{\tau}^w) W_{t+s}(j) h_{t+s}(j) + R_{t+s-1}B_{t+s} + Pr_{t+s} + T_{t+s}$$
$$h_{t+s}(j) = \left(\frac{W_{t+s}(j)}{W_{t+s}}\right)^{-\dfrac{\lambda^w}{\lambda^w-1}} h_{t+s}$$

$$W_{t+s}(j) = \tilde{W}_t(j) \left(\prod_{l=1}^{s} \Pi_{t+l-1}^{\iota^w} \bar{\Pi}^{1-\iota^w} \right) \tag{93}$$

Where $\bar{\tau}^w$ is the subsidy to households who supply differentiated labor varieties. We assume $\bar{\tau}^w = \lambda^w - 1$ to eliminate the distortions due to monopolistic competition among households.

B NK model with search and matching frictions: linear model

This section derives the linear model that approximates the NK model with search and matching model. We first derive the elasticity of labor market tightness with respect to shocks to understand the amplification of shocks in the presence of search and matching frictions. Subsequently, we show that the linear system of the NK model with search and matching frictions can be stated in terms of three equations. For simplicity, we abstract from price indexation and consumption habits from here on.

B.1 Simple analytics

To learn about the amplification of shocks in the framework with endogenous labor supply, we combine the wage bargaining equations to derive an expression for labor market tightness, θ_t.

Substituting the surplus sharing rule, $J_t = \frac{1-\xi}{\xi} H_t$ into the definition of the household's marginal value of employment

$$\begin{aligned} \xi J_t &= -(1-\xi) \frac{\phi_0}{1+\phi} h_t^{1+\phi} \frac{1}{\lambda_t} + (1-\xi)(w_t h_t - b^u) \\ &\quad + \xi (1-\rho) \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} (1 - s_{t+1}) J_{t+1} \right). \end{aligned} \tag{94}$$

Combining with the marginal value of employment to the firm to eliminate the wage rate

$$\begin{aligned} & J_t + (1-\xi) \frac{\phi_0}{1+\phi} h_t^{1+\phi} \frac{1}{\lambda_t} + (1-\xi) b^u \\ = \quad & (1-\xi)\, mpl_t h_t mc_t + (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} (1 - \xi s_{t+1}) J_{t+1} \right] \end{aligned} \tag{95}$$

or recognizing that efficient bargaining over hours worked implies that the marginal product of labor is equal to the marginal rate of substitution between labor and consumption

$$J_t + (1-\xi)\, b^u = (1-\xi)\, \frac{\phi}{1+\phi} mpl_t h_t mc_t + (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} (1-\xi s_{t+1})\, J_{t+1}\right]. \quad (96)$$

Applying the definitions for s_t and q_t, and the condition

$$J_t = \frac{\kappa^v}{q_t} \quad (97)$$

we finally summarize the equations characterizing the wage bargaining process in a single equation

$$\begin{aligned} & \frac{\kappa^v}{\chi} \theta_t^\zeta + (1-\xi)\, b^u \\ = \quad & (1-\xi)\, \frac{\phi}{1+\phi} mpl_t h_t mc_t + (1-\rho) E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(1-\xi \chi \theta_{t+1}^{1-\zeta}\right) \left(\frac{\kappa^v}{\chi} \theta_{t+1}^\zeta\right)\right] \end{aligned} \quad (98)$$

thereby eliminating H_t, J_t, w_t from the system of relevant equations.

In its log-linear form, the expression reduces to

$$\begin{aligned} & \zeta \frac{\kappa^v}{\chi} \theta_{ss}^\zeta \hat{\theta}_t - (1-\rho)\beta \left(\frac{\zeta \kappa^v}{\chi} \theta_{ss}^\zeta - \xi \kappa^v \theta_{ss}\right) E_t \hat{\theta}_{t+1} \\ = \quad & (1-\xi) \frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss} \widehat{mpl}_t + (1-\xi) \frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss} \hat{h}_t \\ & + (1-\xi) \frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss} \hat{mc}_t \\ & + (1-\rho)\beta \left(1-\xi \chi \theta_{ss}^{1-\zeta}\right) \left(\frac{\kappa^v}{\chi} \theta_{ss}^\zeta\right) E_t \left[\hat{\lambda}_{t+1} - \hat{\lambda}_t\right] \end{aligned} \quad (99)$$

and after using the steady state relationship $q_{ss} = \chi \theta_{ss}^{-\zeta}$

$$\begin{aligned} & \zeta \frac{\kappa^v}{q_{ss}} \hat{\theta}_t - (1-\rho)\beta \left(\frac{\zeta \kappa^v}{q_{ss}} - \xi \kappa^v \theta_{ss}\right) E_t \hat{\theta}_{t+1} \\ = \quad & (1-\xi) \frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss} \left(\widehat{mpl}_t + \hat{h}_t + \hat{mc}_t\right) \\ & + (1-\rho)\beta \left(1-\xi q_{ss} \theta_{ss}\right) \left(\frac{\kappa^v}{q_{ss}}\right) E_t \left[\hat{\lambda}_{t+1} - \hat{\lambda}_t\right]. \end{aligned} \quad (100)$$

To simplify the expression in equation (100), note that in the steady state equation (98) implies

$$\left(\frac{\kappa^v}{q_{ss}}\right) \left[1 - (1-\rho)\beta \left(1-\xi q_{ss} \theta_{ss}\right)\right] = (1-\xi) \left[\frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss} - b^u\right]. \quad (101)$$

Using the conditions involving the marginal value of employment to the firm J_t evaluated in the steady state and defining the replacement ratio as $b^u = r^u w_{ss} h_{ss}$

$$b^u = r^u w_{ss} h_{ss} = r^u \left(mpl_{ss} h_{ss} mc_{ss} - (1-(1-\rho)\beta) \frac{\kappa^v}{q_{ss}} \right). \tag{102}$$

Combining equations (101) and (102) defines the bargaining weight ξ in terms of the replacement ratio r^u and other parameters and steady state targets

$$\begin{aligned} & \left(\frac{\kappa^v}{q_{ss}}\right) [1-(1-\rho)\beta (1-\xi q_{ss}\theta_{ss})] \\ = \quad & (1-\xi) \left[\left(\frac{\phi}{1+\phi} - r^u \right) mpl_{ss} h_{ss} mc_{ss} + r^u (1-(1-\rho)\beta) \left(\frac{\kappa^v}{q_{ss}}\right) \right]. \end{aligned} \tag{103}$$

Assuming that changes in variables are small between two periods, we can approximate the response of labor market tightness as

$$\hat{\theta}_t \approx \frac{1}{\Upsilon} \frac{\frac{\phi}{1+\phi} mpl_{ss} h_{ss} mc_{ss}}{\left[\left(\frac{\phi}{1+\phi} - r^u \right) mpl_{ss} h_{ss} mc_{ss} + r^u (1-(1-\rho)\beta) \left(\frac{\kappa^v}{q_{ss}}\right) \right]} \left(\widehat{mpl}_t + \hat{h}_t + \widehat{mc}_t \right) \tag{104}$$

where

$$\Upsilon = \zeta + \frac{(1-\rho)\beta\xi q_{ss}\theta_{ss} (1-\zeta)}{[1-(1-\rho)\beta (1-\xi q_{ss}\theta_{ss})]}. \tag{105}$$

B.2 Implications of negotiating over hours worked

In NK model with search and matching frictions and flexible hours worked, equation (77) resembles its counterpart in the standard NK model with flexible wages. Noticing that

$$c_t = y_t - \kappa^v v_t + b^u (1-n_t) \tag{106}$$

$$\Omega_t^y y_t = a_t n_t h_t \tag{107}$$

where Ω_t^y measures the dispersion of prices, negotiation over hours worked implies

$$\phi_0 \left(\frac{\Omega_t^y y_t}{n_t} \right)^{\phi} (y_t - \kappa^v v_t + b^u (1-n_t))^{\sigma} = \frac{P_t^w}{P_t} a_t^{1+\phi}. \tag{108}$$

In the model with a Walrasian labor market (as in the standard NK model), n_t is constant and search costs are zero

$$\phi_0 \left(\Omega_t^y y_t \right)^{\phi} (y_t)^{\sigma} = \frac{P_t^w}{P_t} a_t^{1+\phi}. \tag{109}$$

Relative to the standard NK model, we need to take into account the dynamics of n_t, v_t, and q_t. Or after log-linearizing, the two different models imply

$$(\phi + \sigma)\,\hat{y}_t = \widehat{\left[\frac{P^w}{P}\right]}_t + (1+\phi)\,\hat{a}_t \tag{110}$$

compared to

$$\left(\phi + \sigma \frac{\frac{y_{ss}}{y_{ss} + b^u(1-n_{ss})}}{1 - \frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})}}\right)\hat{y}_t - \Theta_t = \widehat{\left[\frac{P^w}{P}\right]}_t + (1+\phi)\,\hat{a}_t \tag{111}$$

with the correction term Θ_t being defined as

$$\Theta_t = \left(\phi + \sigma \frac{\frac{b^u n_{ss}}{y_{ss} + b^u(1-n_{ss})}}{1 - \frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})}}\right)\hat{n}_t + \sigma \frac{\frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})}}{1 - \frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})}}\hat{v}_t. \tag{112}$$

The variables $\hat{v}_t$ and $\hat{q}_t$ can be expressed in terms of $\hat{n}_t$ using the (log-linearized) equations that describe the labor market

$$\hat{v}_t = \hat{\theta}_t + \hat{u}_t \tag{113}$$

$$\hat{q}_t = -\zeta\hat{\theta}_t \tag{114}$$

$$\hat{u}_t = -\frac{(1-\rho)n_{ss}}{1-(1-\rho)n_{ss}}\hat{n}_{t-1} \tag{115}$$

$$\hat{n}_t = (1-\rho)\hat{n}_{t-1} + \rho\hat{m}_t \tag{116}$$

$$\hat{m}_t = \hat{u}_t + (1-\zeta)\hat{\theta}_t \tag{117}$$

and therefore

$$\hat{v}_t = \nu_1\hat{n}_t - \nu_2\left(\nu_1 + \frac{n_{ss}}{1-n_{ss}}\right)\hat{n}_{t-1} \tag{118}$$

$$\hat{q}_t = -\zeta\nu_1\hat{n}_t + \zeta\nu_1\nu_2\hat{n}_{t-1} \tag{119}$$

$$\begin{aligned}\hat{\theta}_t &= \frac{1}{\rho(1-\zeta)}\hat{n}_t - \frac{1}{\rho(1-\zeta)}\frac{(1-\rho)(1-n_{ss})}{1-(1-\rho)n_{ss}}\hat{n}_{t-1} \\ &= \nu_1\hat{n}_t - \nu_1\nu_2\hat{n}_{t-1}.\end{aligned} \tag{120}$$

Thus,

$$\begin{aligned}\Theta_t &= \left[\phi + \sigma\frac{\varpi^{b^u}}{1-\kappa^c} + \sigma\frac{\kappa^c}{1-\kappa^c}\nu_1\right]\hat{n}_t - \sigma\frac{\kappa^c}{1-\kappa^c}\nu_1\nu_2\left(1 + \frac{n_{ss}}{1-n_{ss}}\frac{1}{\nu_1}\right)\hat{n}_{t-1} \\ &= \theta_1\hat{n}_t + \theta_2\hat{n}_{t-1}\end{aligned} \tag{121}$$

where

$$\nu_1 = \frac{1}{\rho(1-\zeta)} \tag{122}$$

$$\nu_2 = \frac{(1-\rho)(1-n_{ss})}{1-(1-\rho)n_{ss}} \tag{123}$$

$$\kappa^c = \frac{\kappa^v v_{ss}}{y_{ss}+b^u(1-n_{ss})} \tag{124}$$

$$\varpi^{b^u} = \frac{b^u n_{ss}}{y_{ss}+b^u(1-n_{ss})} \tag{125}$$

$$\varpi^{y_{ss}} = \frac{y_{ss}}{y_{ss}+b^u(1-n_{ss})} \tag{126}$$

$$\theta_1 = \left[\phi + \sigma\frac{\varpi^{b^u}}{1-\kappa^c} + \sigma\frac{\kappa^c}{1-\kappa^c}\nu_1\right] \tag{127}$$

$$\theta_2 = -\sigma\frac{\kappa^c}{1-\kappa^c}\nu_1\nu_2\left(1+\frac{n_{ss}}{1-n_{ss}}\frac{1}{\nu_1}\right) \tag{128}$$

The dynamics of real marginal costs satisfy

$$\hat{mc}_t = \widehat{\left[\frac{P^w}{P}\right]}_t = \left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}\right)\hat{y}_t - (1+\phi)\,\hat{a}_t - (\theta_1\hat{n}_t + \theta_2\hat{n}_{t-1})\,. \tag{129}$$

B.3 Implications of negotiating over the real wage

Combining the first order conditions of the firm with the bargaining outcome over wages, we arrive at the following relationship between real marginal costs of the wholesale retailers and labor market tightness

$$\begin{aligned} & (1-\xi)\,\frac{\phi}{1+\phi}\frac{P^w_t}{P_t}a_t h_t \\ = \quad & (1-\xi)\,b^u + \frac{\kappa^v}{\chi}\theta_t^{\zeta} - (1-\rho)E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1-\chi\theta_{t+1}^{1-\zeta}\right)\left(\frac{\kappa^v}{\chi}\theta_{t+1}^{\zeta}\right)\right]. \end{aligned} \tag{130}$$

Log-linearizing therefore delivers the following relationship between real marginal costs of the retailers and labor market tightness:

$$\begin{aligned} \widehat{\left[\frac{P^w}{P}\right]}_t + \hat{y}_t - \hat{n}_t \quad = \quad & \frac{\zeta}{\mu}\frac{\kappa^v}{q_{ss}}\hat{\theta}_t - \frac{(1-\rho)\beta}{\mu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)E_t\hat{\theta}_{t+1} \\ & + \frac{(1-\rho)\beta}{\mu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)E_t\left[i_t - \pi_{t+1}\right] \end{aligned} \tag{131}$$

where we have used the fact that $\hat{y}_t - \hat{n}_t = \hat{a}_t + \hat{h}_t$ and we defined

$$\mu = (1-\xi)\frac{\phi}{1+\phi}\left[\frac{P^w}{P}\right]_{ss} a_{ss}h_{ss}. \tag{132}$$

Absent flexible hours worked, i.e., $\hat{h}_t = 0$, the above expression is used to substitute out for real marginal costs in the New Keynesian Phillips Curve, see Ravenna and Walsh (2011). Given the movements in marginal costs and the real interests rate, labor market tightness and therefore employment are pinned down.

In the case of flexible hours worked, we can combine equations (111) and (131) to

$$\begin{aligned}(1+\theta_1)\hat{n}_t + \theta_2\hat{n}_{t-1} \quad = \quad & \left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c} + 1\right)\hat{y}_t - (1+\phi)\,\hat{a}_t \\ & -\frac{\zeta}{\mu}\frac{\kappa^v}{q_{ss}}\hat{\theta}_t + \frac{(1-\rho)\beta}{\mu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)E_t\hat{\theta}_{t+1} \\ & -\frac{(1-\rho)\beta}{\mu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)E_t\left[i_t - \pi_{t+1}\right]. \end{aligned} \tag{133}$$

and after substituting out for $\hat{\theta}_t$

$$\begin{aligned} & -\frac{(1-\rho)\beta}{\mu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)\nu_1 E_t\hat{n}_{t+1} \\ & \left[(1+\theta_1) + \frac{\zeta}{\mu}\frac{\kappa^v}{q_{ss}}\nu_1 + \frac{(1-\rho)\beta}{\mu}\left(\frac{\zeta\kappa^v}{q_{ss}} - \xi\kappa^v\theta_{ss}\right)\nu_1\nu_2\right]\hat{n}_t \\ & + \left[\theta_2 - \frac{\zeta}{\mu}\frac{\kappa^v}{q_{ss}}\nu_1\nu_2\right]\hat{n}_{t-1} \\ = \quad & \left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c} + 1\right)\hat{y}_t - (1+\phi)\,\hat{a}_t \\ & -\frac{(1-\rho)\beta}{\mu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)E_t\left[i_t - \pi_{t+1}\right]. \end{aligned} \tag{134}$$

B.4 Aggregate demand equation

By taking account home production (i.e. unemployment benefits are not financed from any resources), the resource constraint in the economy is

$$c_t = y_t - \kappa^v v_t + b^u(1-n_t). \tag{135}$$

Log-linearizing delivers

$$\hat{c}_t = \frac{\varpi^{y_{ss}}}{1-\kappa^c}\hat{y}_t - \frac{\theta_1-\phi}{\sigma}\hat{n}_t + \frac{\theta_2}{\sigma}\hat{n}_{t-1} \tag{136}$$

combined with log-linearized Euler equation for holding bonds

$$-\sigma\left(\hat{c}_t - \hat{c}_{t+1}\right) = i_t - E_t\pi_{t+1} \tag{137}$$

we have the log-linearized aggregate demand equation

$$\begin{aligned}\hat{y}_t &= E_t\hat{y}_{t+1} - \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left(i_t - E_t\pi_{t+1}\right) \\ &\quad -\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left[\left(\theta_1-\phi\right)\left(E_t\hat{n}_{t+1}-\hat{n}_t\right)+\theta_2\left(\hat{n}_t-\hat{n}_{t-1}\right)\right]. \qquad (138)\end{aligned}$$

B.5 Linear model

The policy rule not withstanding, the linear NK model with search and matching frictions is summarized by the following three equations

$$\begin{aligned}\pi_t &= \beta E_t\pi_{t+1} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\Big[\left(\phi+\sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}\right)\hat{y}_t \\ &\quad -\left(1+\phi\right)\hat{a}_t - \left(\theta_1\hat{n}_t+\theta_2\hat{n}_{t-1}\right)\Big] + \hat{\theta}_{p,t} \qquad (139)\\ \hat{y}_t &= E_t\hat{y}_{t+1} - \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left(i_t-E_t\pi_{t+1}\right) \\ &\quad -\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left[\left(\theta_1-\phi\right)\left(E_t\hat{n}_{t+1}-\hat{n}_t\right)+\theta_2\left(\hat{n}_t-\hat{n}_{t-1}\right)\right] \qquad (140)\\ \gamma_1 E_t\hat{n}_{t+1}+\gamma_2\hat{n}_t+\gamma_3\hat{n}_{t-1} &= \left(\phi+\sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}+1\right)\hat{y}_t - \left(1+\phi\right)\hat{a}_t \\ &\quad -\frac{(1-\rho)\beta}{\mu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)E_t\left[i_t-\pi_{t+1}\right] \qquad (141)\end{aligned}$$

with the coefficients

$$\gamma_1 = -\frac{(1-\rho)\beta}{\mu}\left(\frac{\zeta\kappa^v}{q_{ss}}-\xi\kappa^v\theta_{ss}\right)\nu_1 \qquad (142)$$

$$\gamma_2 = \left[(1+\theta_1)+\frac{\zeta}{\mu}\frac{\kappa^v}{q_{ss}}\nu_1+\frac{(1-\rho)\beta}{\mu}\left(\frac{\zeta\kappa^v}{q_{ss}}-\xi\kappa^v\theta_{ss}\right)\nu_1\nu_2\right] \qquad (143)$$

$$\gamma_3 = \left[\theta_2-\frac{\zeta}{\mu}\frac{\kappa^v}{q_{ss}}\nu_1\nu_2\right] \qquad (144)$$

$$\kappa^p = \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}. \qquad (145)$$

According to the NKPC (139), similar to the standard NK model, price inflation dynamics in search and matching models are determined by current and future real marginal costs which in turn are related to the ratio of real wage to marginal product of labor. However, the real wage in search and matching models is determined through a bargaining process rather than simply equal to marginal rate of substitution between leisure and consumption. Thus, labor market variables affect inflation dynamics directly through the NKPC. Furthermore, the real interest rate affects inflation dynamics the third equation (141). Ravenna and Walsh (2011) refers to this channel, which is absent in standard NK model, as the "cost-channel". In contrast to standard NK model, the aggregate demand equation (140) in search and matching models features not only forward looking behavior but also backward looking behavior even with standard household preferences that exhibit no habit persistence.

The standard NK model and the model in Ravenna and Walsh (2011) arise as special cases:

- Absent labor market frictions, $\hat{n}_t = 0$, $\kappa^c = 0$, $\varpi^{y_{ss}} = 1$ and equation (141) is taken out of the model, the standard NK model with flexible wages reemerges

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}(\phi+\sigma)\left(\hat{y}_t - \frac{(1+\phi)}{(\phi+\sigma)}\hat{a}_t\right) \tag{146}$$

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}). \tag{147}$$

- If there exists labor market frictions, but the individual labor supply is completely inelastic as in Ravenna and Walsh (2011), i.e. $\phi = \infty$, equation (141) reduces to $\hat{y}_t = \hat{a}_t + \hat{n}_t$. After substituting out for $\hat{y}_t$ by $\hat{n}_t$ in the aggregate demand equation, we obtain

$$\begin{aligned}\pi_t &= \beta E_t\pi_{t+1} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\Bigg[\gamma_1 E_t\hat{n}_{t+1} + (\gamma_2-\theta_1)\hat{n}_t + (\gamma_3-\theta_2)\hat{n}_{t-1} \\ &\quad -\hat{a}_t + \frac{(1-\rho)\beta}{\mu}(1-\xi q_{ss}\theta_{ss})\left(\frac{\kappa^v}{q_{ss}}\right)E_t(i_t-\pi_{t+1})\Bigg] \end{aligned} \tag{148}$$

$$\hat{n}_t = \gamma_n E_t\hat{n}_{t+1} + (1-\gamma_n)\hat{n}_{t-1} - \gamma_n^d \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}(i_t - E_t\pi_{t+1}) + \gamma_n^d(\rho_a - 1)\hat{a}_t \tag{149}$$

where

$$\gamma_n = \frac{\varpi^{y_{ss}}\sigma - (1-\kappa^c)(\theta_1-\phi)}{\varpi^{y_{ss}}\sigma + (1-\kappa^c)(\theta_2-\theta_1+\phi)} \tag{150}$$

$$\gamma_n^d = \frac{\varpi^{y_{ss}}\sigma}{\varpi^{y_{ss}}\sigma + (1-\kappa^c)(\theta_2-\theta_1+\phi)}. \tag{151}$$

C Optimal targeting rule for NK model with search and matching frictions

Having obtained the (linear) equations that describe the behavior of the private sector, we still need to derive the objective function of the policymaker as a purely quadratic approximation to the preferences of the representative household to formulate the linear-quadratic problem from which we derive the optimal targeting rule in the NK model with search and matching frictions. In this section, we first derive the correct quadratic loss function as the approximation to the preferences of the representative household. Then we obtain the first order conditions associated with the policymaker's problem of optimizing the (quadratic) objective function subject to the (linear) equations that describe the behavior of the private sector. Finally, the optimal targeting rule is then derived by combining the first order conditions to the policymaker's problem into a single equation without Lagrange multipliers.

C.1 Simplified nonlinear optimality conditions

Before retrieving a numerical representation of the quadratic loss function, we write the nonlinear model in terms of the variables that also enter the set of log-linear equations $\left\{n_t, i_t, y_t, \Pi_t\right\}$ as well as the variables $\left\{U_t^p, V_t^p, \Omega_t^p, \tilde{p}_t^{opt}\right\}$.

The number of job seekers is already expressed in terms of employment only

$$u_t = 1 - (1-\rho)n_{t-1} \tag{152}$$

and matches evolve thus according to

$$m_t = n_t - (1-\rho)n_{t-1}. \tag{153}$$

Using the matching technology $m_t = \chi u_t^{\zeta} v_t^{1-\zeta}$, the total number of vacancies satisfies

$$v_t = \left(\frac{m_t}{\chi u_t^{\zeta}}\right)^{\frac{1}{1-\zeta}} = \chi^{-\frac{1}{1-\zeta}} \left(n_t - (1-\rho)n_{t-1}\right)^{\frac{1}{1-\zeta}} \left(1-(1-\rho)n_{t-1}\right)^{-\frac{\zeta}{1-\zeta}} \tag{154}$$

while labor market tightness can be shown to follow

$$\theta_t = \frac{v_t}{u_t} = \chi^{-\frac{1}{1-\zeta}} \left(n_t - (1-\rho)n_{t-1}\right)^{\frac{1}{1-\zeta}} \left(1-(1-\rho)n_{t-1}\right)^{-\frac{1}{1-\zeta}}. \tag{155}$$

Finally, the vacancy filling rate is given by

$$q_t = \chi\theta_t^{-\zeta} = \chi^{\frac{1}{1-\zeta}} \left(n_t - (1-\rho)n_{t-1}\right)^{-\frac{\zeta}{1-\zeta}} \left(1-(1-\rho)n_{t-1}\right)^{\frac{\zeta}{1-\zeta}} \tag{156}$$

and the job finding rate can be written as

$$s_t = \frac{m_t}{u_t} = \frac{n_t - (1-\rho)n_{t-1}}{1-(1-\rho)n_{t-1}}. \tag{157}$$

Using the production technology, hours worked can expressed as

$$h_t = \frac{\Omega_t^y y_t}{a_t n_t}. \tag{158}$$

The resource constraint implies for consumption that

$$c_t \;\;=\;\; y_t - \kappa^v v_t + b^u(1-n_t). \tag{159}$$

The equation governing vacancy postings (76) can be stated as

$$\left(\frac{\kappa^v}{q_t}\right) c_t^{-\sigma} = (1-\xi)\left(\frac{\phi}{1+\phi}\phi_0 h_t^{1+\phi} - b^u c_t^{-\sigma}\right) + (1-\rho)\beta E_t c_{t+1}^{-\sigma}(1-\xi s_{t+1})\left(\frac{\kappa^v}{q_{t+1}}\right) \tag{160}$$

whereas the wage bargaining equation is

$$w_t h_t = \xi\left(\phi_0 h_t^{1+\phi} c_t^{\sigma} + (1-\rho)\beta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\theta_{t+1}\kappa^v\right) + (1-\xi)\left(b^u + \phi_0 \frac{h_t^{1+\phi}}{1+\phi} c_t^{\sigma}\right). \tag{161}$$

Finally, the nonlinear equations governing the evolution of prices in equilibrium are, the optimal price

$$\tilde{p}_t^{opt} = \frac{U_t^p}{V_t^p} \tag{162}$$

which is computed as the ratio of the recursively defined terms U_t^p and V_t^p

$$U_t^p = \frac{1+\theta_{p,t}}{\theta^p}\phi_0 h_t^{\phi}\frac{y_t}{a_t} + \xi^p \beta E_t \left(\Pi_{t+1}\right)^{\left(\frac{1+\theta^p}{\theta^p}\right)} U_{t+1}^p \tag{163}$$

$$V_t^p = \frac{(1+\bar{\tau}^p)}{\theta^p} y_t c_t^{-\sigma} + \xi^p \beta E_t \left(\Pi_{t+1}\right)^{\frac{1}{\theta^p}} V_{t+1}^p. \tag{164}$$

The definition of the price level implies

$$1 = \xi^p \left(\Pi_t\right)^{\frac{1}{\theta^p}} + (1-\xi^p)\left(\tilde{p}_t^{opt}\right)^{-\frac{1}{\theta^p}} \tag{165}$$

and price dispersion evolves according to

$$\Omega_t^p = \xi^p \left(\Pi_t\right)^{\frac{1+\theta^p}{\theta^p}} \Omega_{t-1}^p + (1-\xi^p)(\tilde{p}_t^{opt})^{-\frac{1+\theta^p}{\theta^p}}. \tag{166}$$

Recall, that we continue to abstract from price indexation and consumption habits.

C.2 Correct quadratic loss function

Following a large body of the literature, we compute the optimal monetary policy under commitment from *the timeless perspective* as the reference point to evaluate the performance of different policies. Optimality from the timeless perspective assumes that the policymaker can "pre-commit" at the beginning of time. This assumption converts the optimal policy problem into a recursive problem with time invariant functions as shown in detail in Benigno and Woodford (2012). As shown in Bodenstein, Guerrieri, and LaBriola (2014), the first-order approximation to the system of first order conditions associated with original nonlinear model can be mapped into the LQ problem

$$\begin{aligned}
&\max_{\{\hat{x}_t\}_{t=t_0}^{\infty}} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2}\, \hat{x}_t' A(L) \hat{x}_t + \hat{x}_t' B(L) \zeta_{t+1}\right] \\
&s.t. \\
&E_t C(L) \hat{x}_{t+1} + D(L) \zeta_t = 0 \\
&C(L) \hat{x}_{t_0} = d_{t_0} \\
&\zeta_t = \Gamma \zeta_{t-1} + \Upsilon \xi_t
\end{aligned} \tag{167}$$

where $\hat{x}_{t_0}$ measures the (log-) deviation of variable "x" from its value assumed in deterministic steady state. The matrices $(A(L), B(L))$ capture the second-order approximation of the welfare function, where "L" denotes the lag-operator. The matrices $C(L)$ and $D(L)$ capture the linear approximation of the constraints. The linear constraints $C(L)\hat{x}_{t_0} = d_{t_0}$ implement the timeless perspective through the appropriate choice of d_{t_0}. The model description is completed by the evolution of the exogenous variables, the last equation in (167). The innovations ξ_t follow *iid* standard normal distributions.

All welfare relevant matrices in the above LQ problem can be retrieved using the numerical approach in Bodenstein, Guerrieri, and LaBriola (2014). After retrieving the welfare matrices $A(L)$ and $B(L)$ and accounting for all zero elements, the approximation to the preferences of the representative household is given by

$$\begin{aligned}\tilde{\mathcal{L}}^{s\&m} = \quad & \frac{1}{2}a_{2,2}\pi_t^2 + \frac{1}{2}a_{3,3}\hat{n}_t^2 + \frac{1}{2}a_{8,8}\hat{y}_t^2 + \frac{1}{2}a_{11,11}\hat{n}_{t-1}^2 + a_{3,8}\hat{n}_t\hat{y}_t + a_{3,11}\hat{n}_t\hat{n}_{t-1} + a_{8,11}\hat{y}_t\hat{n}_{t-1} \\ & + \frac{1}{2}a_{4,4}(\hat{\tilde{p}}_t^{opt})^2 + a_{2,6}\pi_t\hat{U}_t^p + a_{2,7}\pi_t\hat{V}_t^p + a_{6,7}\hat{U}_t^p\hat{V}_t^p + \frac{1}{2}a_{7,7}(\hat{V}_t^p)^2 \\ & + b_{3,3}\hat{n}_t\hat{n}_{t-1} + b_{8,3}\hat{y}_t\hat{n}_{t-1} + c_{3,1}\hat{n}_t\hat{a}_t + c_{3,2}\hat{n}_t\hat{\theta}_{p,t} + c_{8,1}\hat{y}_t\hat{a}_t + c_{8,2}\hat{y}_t\hat{\theta}_{p,t} \qquad (168)\end{aligned}$$

where $\hat{y}_t$ is output, π_t refers to price inflation, and $\hat{n}_t$ stands for employment. $\hat{\tilde{p}}_t^{opt}$ is the optimal price set by re-optimizing firms. $\hat{U}_t^p$ and $\hat{V}_t^p$ are log-linear versions of the variables $\hat{U}_t$ and V_t^p. $\hat{a}_t$ is technology shock and $\hat{\theta}_{p,t}$ is price markup shock. $a_{i,j} = A_0(i,j)$, $b_{i,j} = A_1(i,j)$, and $c_{i,j} = B_1(i,j)$ for corresponding index (i,j) are the entries in $A(L)$ and $B(L)$. Besides terms that are already present in the standard NK model, labor market variables affect the loss function in the search and matching framework. Current and lagged employment enter the approximation.

When using a first order approximation, the nonlinear equations associated with Calvo sticky prices can be summarized in the NKPC for price inflation. Therefore, sticky price variables $\{\hat{\tilde{p}}_t^{opt}, \hat{U}_t^p, \hat{V}_t^p, \hat{\Omega}_t^p\}$ will only show up in the nonlinear system but not in the log-linearized system. To make the loss function work correspond to the linear structural equations, these sticky price variables have to be substituted out.

Log-linearizing the equation (165) delivers

$$\hat{\tilde{p}}_t^{opt} = \frac{\xi^p}{1-\xi^p}\pi_t. \qquad (169)$$

Equation (169) can be used to substitute out $\hat{\tilde{p}}_t^{opt}$ in the loss function.

Log-linearizing the equation describing the evolution of price dispersion (166) provides

$$\begin{aligned}\hat{\Omega}_t^p \quad & = \xi^p\hat{\Omega}_{t-1}^p + \xi^p\frac{1+\theta^p}{\theta^p}\pi_t - (1-\xi^p)\frac{1+\theta^p}{\theta^p}\hat{\tilde{p}}_t^{opt} \\ & = \xi^p\hat{\Omega}_{t-1}^p. \qquad (170)\end{aligned}$$

Thus, price dispersion can be ignored to the first order.

Applying the log-linearization for equations (162) and (165), we have

$$\begin{aligned}& a_{2,6}\pi_t\hat{U}_t^p + a_{2,7}\pi_t\hat{V}_t^p + a_{6,7}\hat{U}_t^p\hat{V}_t^p + \frac{1}{2}a_{7,7}(\hat{V}_t^p)^2 \\ & = a_{2,6}\pi_t\left(\hat{\tilde{p}}_t^{opt} + \hat{V}_t^p\right) + a_{2,7}\pi_t\hat{V}_t^p + a_{6,7}\left(\hat{\tilde{p}}_t^{opt} + \hat{V}_t^p\right)\hat{V}_t^p + \frac{1}{2}a_{7,7}(\hat{V}_t^p)^2 \\ & = a_{2,6}\pi_t\hat{\tilde{p}}_t^{opt} + a_{2,6}\pi_t\hat{V}_t^p + a_{2,7}\pi_t\hat{V}_t^p + a_{6,7}\hat{V}_t^p\hat{\tilde{p}}_t^{opt} + \left(a_{6,7} + \frac{1}{2}a_{7,7}\right)(\hat{V}_t^p)^2\end{aligned}$$

$$
\begin{aligned}
&= a_{2,6}\pi_t \hat{\tilde{p}}_t^{opt} + a_{2,6}\pi_t \hat{V}_t^p + a_{2,7}\pi_t \hat{V}_t^p + a_{6,7}\hat{V}_t^p \hat{\tilde{p}}_t^{opt} \\
&= a_{2,6}\frac{\xi^p}{1-\xi^p}\pi_t^2 + \left(a_{2,6} + a_{2,7} + a_{6,7}\frac{\xi^p}{1-\xi^p}\right)\pi_t \hat{V}_t^p \\
&= a_{2,6}\frac{\xi^p}{1-\xi^p}\pi_t^2 \qquad (171)
\end{aligned}
$$

The first identity comes from the relationship $\hat{U}_t^p = \hat{\tilde{p}}_t^{opt} + \hat{V}_t^p$; the third identity is true as $a_{6,7} + \frac{1}{2}a_{7,7} = 0$; plugging in equation (169) gives us the fourth identity; the fifth identity holds as $a_{2,6} + a_{2,7} + a_{6,7}\frac{\xi^p}{1-\xi^p} = 0$.

We convert the approximation to household preferences, $\tilde{\mathcal{L}}_t^{s\&m}$, into a loss function by defining $\mathcal{L}_t^{s\&m} = -\tilde{\mathcal{L}}_t^{s\&m}$. The loss function in the search and matching model is therefore written as

$$
\begin{aligned}
\mathcal{L}_t^{s\&m} \;=\; & P_{\pi,\pi}\pi_t^2 + P_{y,y}\hat{y}_t^2 + P_{n,n}\hat{n}_t^2 + P_{n^-,n^-}\hat{n}_{t-1}^2 + P_{y,n}\hat{n}_t\hat{y}_t + P_{y,n^-}\hat{y}_t\hat{n}_{t-1} \\
& + P_{n,n^-}\hat{n}_t\hat{n}_{t-1} + P_{n,a}\hat{n}_t\hat{a}_t + P_{n,p}\hat{n}_t\hat{\theta}_{p,t} + P_{y,a}\hat{y}_t\hat{a}_t + P_{y,p}\hat{y}_t\hat{\theta}_{p,t} \qquad (172)
\end{aligned}
$$

where

$$
\begin{aligned}
P_{\pi,\pi} &= -\frac{1}{2}a_{2,2} - \frac{1}{2}a_{4,4}\left(\frac{\xi^p}{1-\xi^p}\right)^2 - a_{2,6}\frac{\xi^p}{1-\xi^p} \\
P_{y,y} &= -\frac{1}{2}a_{8,8} \\
P_{n,n} &= -\frac{1}{2}a_{3,3} \\
P_{n^-,n^-} &= -\frac{1}{2}a_{11,11} \\
P_{y,n} &= -\frac{1}{2}a_{3,8} \\
P_{y,n^-} &= -\left(a_{8,11} + b_{8,3}\right) \\
P_{n,n^-} &= -\left(a_{3,11} + b_{3,3}\right) \\
P_{n,a} &= -c_{3,1} \\
P_{n,p} &= -c_{3,2} \\
P_{y,a} &= -c_{8,1} \\
P_{y,p} &= -c_{8,2}.
\end{aligned}
$$

To sum up, the procedure for deriving the correct loss function in the search and matching models involves:

1. deriving the nonlinear equilibrium conditions for the original model;

2. simplifying the nonlinear system of equations such that it only involves variables that show up in the log-linearized model, together with sticky price variables $\{U^p, V^p, \tilde{p}_t^{opt}, \Omega^y\}$;
3. applying the numerical approach to retrieve welfare matrices based on the simplified equation system;
4. writing out the loss function by plugging in retrieved welfare matrices;
5. using the log-linearized structural equations to eliminate the sticky price variables $\{U^p, V^p, \tilde{p}_t^{opt}, \Omega^y\}$ in the loss function.
6. obtaining the correct quadratic loss function, even though we can only know numerically the values of the coefficients which in turn depend on the model's structural parameters.

C.3 First order conditions of the LQ problem

The correct LQ system is given by

$$\min_{\{\pi_t, i_t, \hat{n}_t, \hat{y}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ P_{\pi,\pi}\pi_t^2 + P_{y,y}\hat{y}_t^2 + P_{n,n}\hat{n}_t^2 + P_{n^-,n^-}\hat{n}_{t-1}^2 + P_{y,n}\hat{n}_t\hat{y}_t$$
$$+P_{y,n^-}\hat{y}_t\hat{n}_{t-1} + P_{n,n^-}\hat{n}_t\hat{n}_{t-1} + P_{n,a}\hat{n}_t\hat{a}_t + P_{n,p}\hat{n}_t\hat{\theta}_{p,t} + P_{y,a}\hat{y}_t\hat{a}_t + P_{y,p}\hat{y}_t\hat{\theta}_{p,t} \Bigg\}$$

$s.t.$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p} \Bigg[\left(\phi + \sigma \frac{\varpi^{y_{ss}}}{1-\kappa^c} \right) \hat{y}_t$$
$$- (1+\phi)\,\hat{a}_t - (\theta_1 \hat{n}_t + \theta_2 \hat{n}_{t-1}) \Bigg] + \hat{\theta}_{p,t}$$
$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}(i_t - E_t\pi_{t+1})$$
$$-\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\left[(\theta_1-\phi)\left(E_t\hat{n}_{t+1} - \hat{n}_t\right) + \theta_2\left(\hat{n}_t - \hat{n}_{t-1}\right)\right]$$
$$\gamma_1 E_t \hat{n}_{t+1} + \gamma_2 \hat{n}_t + \gamma_3 \hat{n}_{t-1} = \left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c} + 1\right)\hat{y}_t - (1+\phi)\,\hat{a}_t$$
$$-\frac{(1-\rho)\beta}{\mu}\left(1 - \xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right) E_t\left[i_t - \pi_{t+1}\right]. \tag{173}$$

The problem is to minimize the quadratic objective function subject to linear structural equations.

Taking first order conditions delivers

$$(i_t): \qquad \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\Lambda_{2,t} + \frac{(1-\rho)\beta}{\mu}\left(1 - \xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)\Lambda_{3,t} = 0 \tag{174}$$

$$(\pi_t): \quad 2P_{\pi,\pi}\pi_t + \Lambda_{1,t} - \Lambda_{1,t-1} - \frac{1}{\beta}\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\Lambda_{2,t-1}$$
$$-\frac{1}{\beta}\frac{(1-\rho)\beta}{\mu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)\Lambda_{3,t-1} = 0 \tag{175}$$
$$(\hat{y}_t): \quad 2P_{y,y}\hat{y}_t + P_{y,n}\hat{n}_t + P_{y,n^-}\hat{n}_{t-1} + P_{y,a}\hat{a}_t + P_{y,p}\hat{\theta}_{p,t}$$
$$-\frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c}\right)\Lambda_{1,t} + \Lambda_{2,t} - \frac{1}{\beta}\Lambda_{2,t-1}$$
$$-\left(\phi + \sigma\frac{\varpi^{y_{ss}}}{1-\kappa^c} + 1\right)\Lambda_{3,t} = 0 \tag{176}$$
$$(\hat{n}_t): \quad 2P_{n,n}\hat{n}_t + 2\beta P_{n^-,n^-}\hat{n}_t + P_{y,n}\hat{y}_t + \beta P_{y,n^-}E_t\hat{y}_{t+1} + P_{n,n^-}\hat{n}_{t-1}$$
$$+\beta P_{n,n^-}E_t\hat{n}_{t+1} + P_{n,a}\hat{a}_t + P_{n,p}\hat{\theta}_{p,t} + \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\theta_1\Lambda_{1,t}$$
$$+\beta\frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p}\theta_2 E_t\Lambda_{1,t+1} + \frac{1}{\varpi^{y_{ss}}}\frac{1}{\beta}\frac{1-\kappa^c}{\sigma}(\theta_1-\phi)\Lambda_{2,t-1}$$
$$-\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}(\theta_1-\phi)\Lambda_{2,t} + \frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}\theta_2\Lambda_{2,t} - \frac{1}{\varpi^{y_{ss}}}\beta\frac{1-\kappa^c}{\sigma}\theta_2 E_t\Lambda_{2,t+1}$$
$$+\frac{1}{\beta}\gamma_1\Lambda_{3,t-1} + \gamma_2\Lambda_{3,t} + \beta\gamma_3 E_t\Lambda_{3,t+1} = 0 \tag{177}$$

C.4 Substituting out Lagrange multipliers and the optimal targeting rule

To simplify notation, we define

$$\begin{aligned}
\varnothing_1 &= \phi + \frac{\varpi^{y_{ss}}\sigma}{1-\kappa^c} \\
\kappa^p &= \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p} \\
\nu^\Lambda &= -\frac{\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}}{\frac{(1-\rho)\beta}{\mu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}}\right)} \\
G_t^\Lambda &= 2P_{y,y}\hat{y}_t + P_{y,n}\hat{n}_t + P_{y,n^-}\hat{n}_{t-1} + P_{y,a}\hat{a}_t + P_{y,p}\hat{\theta}_{p,t} \\
H_t^\Lambda &= 2P_{n,n}\hat{n}_t + 2\beta P_{n^-,n^-}\hat{n}_t + P_{y,n}\hat{y}_t + \beta P_{y,n^-}E_t\hat{y}_{t+1} + P_{n,n^-}\hat{n}_{t-1} + \beta P_{n,n^-}E_t\hat{n}_{t+1} \\
&\quad + P_{n,a}\hat{a}_t + P_{n,p}\hat{\theta}_{p,t}.
\end{aligned}$$

Then the set of first order conditions simplifies to

$$(i_t): \quad \Lambda_{3,t} = \nu^\Lambda\Lambda_{2,t} \tag{178}$$
$$(\pi_t): \quad 2P_{\pi,\pi}\pi_t + \Lambda_{1,t} - \Lambda_{1,t-1} = 0 \tag{179}$$
$$(\hat{y}_t): \quad G_t^\Lambda - \kappa^p\varnothing_1\Lambda_{1,t} + \Lambda_{2,t} - \frac{1}{\beta}\Lambda_{2,t-1} - (1+\varnothing_1)\Lambda_{3,t} = 0 \tag{180}$$

$$(\hat{n}_t): \quad H_t^\Lambda + \kappa^p\theta_1\Lambda_{1,t} + \kappa^p\beta\theta_2 E_t\Lambda_{1,t+1} + \frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi)\Lambda_{2,t-1} + \frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right)\Lambda_{2,t}$$
$$-\frac{1}{\o_1-\phi}\beta\theta_2 E_t\Lambda_{2,t+1} + \frac{1}{\beta}\gamma_1\Lambda_{3,t-1} + \gamma_2\Lambda_{3,t} + \beta\gamma_3 E_t\Lambda_{3,t+1} = 0. \tag{181}$$

Substituting out $\Lambda_{3,t}$ in equation (180) and (181) by using equation (178),

$$G_t^\Lambda - \kappa^p\o_1\Lambda_{1,t} + \left[1-(1+\o_1)\nu^\Lambda\right]\Lambda_{2,t} - \frac{1}{\beta}\Lambda_{2,t-1} = 0 \tag{182}$$

and

$$\begin{aligned}
&H_t^\Lambda + \kappa^p\theta_1\Lambda_{1,t} + \kappa^p\beta\theta_2 E_t\Lambda_{1,t+1} + \frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi)\Lambda_{2,t-1} + \frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right)\Lambda_{2,t} \\
&-\frac{1}{\o_1-\phi}\beta\theta_2 E_t\Lambda_{2,t+1} + \frac{1}{\beta}\gamma_1\nu^\Lambda\Lambda_{2,t-1} + \gamma_2\nu^\Lambda\Lambda_{2,t} + \beta\gamma_3\nu^\Lambda E_t\Lambda_{2,t+1} \\
&= H_t^\Lambda + \kappa^p\theta_1\Lambda_{1,t} + \kappa^p\beta\theta_2 E_t\Lambda_{1,t+1} + \left[\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^\Lambda\right]\Lambda_{2,t-1} \\
&+ \left[\frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right) + \gamma_2\nu^\Lambda\right]\Lambda_{2,t} + \left[\beta\gamma_3\nu^\Lambda - \frac{1}{\o_1-\phi}\beta\theta_2\right]E_t\Lambda_{2,t+1} \\
&= 0.
\end{aligned} \tag{183}$$

Since price inflation is defined as the change in the price level (in terms of deviation from steady state) $\pi_t = \hat{P}_t - \hat{P}_{t-1}$, we can then express $\Lambda_{1,t}$ as proportional to the price level $\hat{P}_t$ from equation (179). At the same time, the equation $\pi_t = \hat{P}_t - \hat{P}_{t-1}$ has to be added into the model system. It is straightforward to show

$$\Lambda_{1,t} = -2P_{\pi,\pi}\hat{P}_t. \tag{184}$$

Plugging the expression of $\Lambda_{1,t}$ into equation (182) and (183),

$$\begin{aligned}
& G_t^\Lambda - \kappa^p\o_1\Lambda_{1,t} + \left[1-(1+\o_1)\nu^\Lambda\right]\Lambda_{2,t} - \frac{1}{\beta}\Lambda_{2,t-1} \\
= \quad & G_t^\Lambda + 2\kappa^p\o_1 P_{\pi,\pi}\hat{P}_t + \left[1-(1+\o_1)\nu^\Lambda\right]\Lambda_{2,t} - \frac{1}{\beta}\Lambda_{2,t-1} \\
= \quad & 0
\end{aligned} \tag{185}$$

and

$$\begin{aligned}
&H_t^\Lambda + \kappa^p\theta_1\Lambda_{1,t} + \kappa^p\beta\theta_2 E_t\Lambda_{1,t+1} + \left[\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^\Lambda\right]\Lambda_{2,t-1} \\
&+ \left[\frac{1}{\o_1-\phi}\left(\theta_2+\phi-\theta_1\right) + \gamma_2\nu^\Lambda\right]\Lambda_{2,t} + \left[\beta\gamma_3\nu^\Lambda - \frac{1}{\o_1-\phi}\beta\theta_2\right]E_t\Lambda_{2,t+1} \\
&= H_t^\Lambda - 2\kappa^p\theta_1 P_{\pi,\pi}\hat{P}_t - 2\kappa^p\beta\theta_2 P_{\pi,\pi}E_t\hat{P}_{t+1} + \left[\frac{1}{\o_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^\Lambda\right]\Lambda_{2,t-1}
\end{aligned}$$

$$
\begin{aligned}
&+\left[\frac{1}{\varnothing_1-\phi}\left(\theta_2+\phi-\theta_1\right)+\gamma_2\nu^{\Lambda}\right]\Lambda_{2,t}+\left[\beta\gamma_3\nu^{\Lambda}-\frac{1}{\varnothing_1-\phi}\beta\theta_2\right]E_t\Lambda_{2,t+1}\\
&=0.
\end{aligned} \tag{186}
$$

We further define

$$
\begin{aligned}
\chi^{\Lambda}_{-1} &= \left[\frac{1}{\varnothing_1-\phi}\frac{1}{\beta}(\theta_1-\phi)+\frac{1}{\beta}\gamma_1\nu^{\Lambda}\right]\\
\chi^{\Lambda}_{0} &= \left[\frac{1}{\varnothing_1-\phi}\left(\theta_2+\phi-\theta_1\right)+\gamma_2\nu^{\Lambda}\right]\\
\chi^{\Lambda}_{1} &= \left[\beta\gamma_3\nu^{\Lambda}-\frac{1}{\varnothing_1-\phi}\beta\theta_2\right]\\
\beta_\delta &= \frac{1}{\beta\left[1-\left(1+\varnothing_1\right)\nu^{\Lambda}\right]}\\
\chi^{\Lambda 2} &= \left(\frac{\chi^{\Lambda}_{-1}}{\beta_\delta}+\chi^{\Lambda}_{0}+\chi^{\Lambda}_{1}\beta_\delta\right).
\end{aligned}
$$

From equation (185), we get

$$
\Lambda_{2,t}-\frac{1}{\beta\left[1-\left(1+\varnothing_1\right)\nu^{\Lambda}\right]}\Lambda_{2,t-1}=-\frac{1}{1-\left(1+\varnothing_1\right)\nu^{\Lambda}}\left(G^{\Lambda}_t+2\kappa^p\varnothing_1P_{\pi,\pi}\hat{P}_t\right) \tag{187}
$$

or

$$
\Lambda_{2,t}-\beta_\delta\Lambda_{2,t-1}=-\beta\beta_\delta\left(G^{\Lambda}_t+2\kappa^p\varnothing_1P_{\pi,\pi}\hat{P}_t\right). \tag{188}
$$

This equation implies an expression for $\Lambda_{2,t}$

$$
\begin{aligned}
&\Lambda_{2,t}\\
&=-\beta\beta_\delta\sum_{s=0}^{\infty}\left(\beta_\delta\right)^s\left(G^{\Lambda}_{t-s}+2\kappa^p\varnothing_1P_{\pi,\pi}\hat{P}_{t-s}\right)\\
&=-\beta\beta_\delta\sum_{s=0}^{\infty}\left(\beta_\delta\right)^s\Big(2P_{y,y}\hat{y}_{t-s}+P_{y,n}\hat{n}_{t-s}+P_{y,n^-}\hat{n}_{t-1-s}+P_{y,a}\hat{a}_{t-s}\\
&\quad+P_{y,p}\hat{\theta}_{p,t-s}+2\kappa^p\varnothing_1P_{\pi,\pi}\hat{P}_{t-s}\Big)\\
&=-\beta\beta_\delta 2P_{y,y}\sum_{s=0}^{\infty}\left(\beta_\delta\right)^s\hat{y}_{t-s}-\beta\beta_\delta P_{y,n}\sum_{s=0}^{\infty}\left(\beta_\delta\right)^s\hat{n}_{t-s}-\beta\beta_\delta P_{y,n^-}\sum_{s=0}^{\infty}\left(\beta_\delta\right)^s\hat{n}_{t-1-s}\\
&\quad-\beta\beta_\delta P_{y,a}\sum_{s=0}^{\infty}\left(\beta_\delta\right)^s\hat{a}_{t-s}-\beta\beta_\delta P_{y,p}\sum_{s=0}^{\infty}\left(\beta_\delta\right)^s\hat{\theta}_{p,t-s}-\beta\beta_\delta 2\kappa^p\varnothing_1P_{\pi,\pi}\sum_{s=0}^{\infty}\left(\beta_\delta\right)^s\hat{P}_{t-s}\\
&=-\beta\beta_\delta 2P_{y,y}\hat{y}^{WA}_t-\beta\beta_\delta P_{y,n}\hat{n}^{WA}_t-\beta\beta_\delta P_{y,n^-}\hat{n}^{WA}_{t-1}-\beta\beta_\delta P_{y,a}\hat{a}^{WA}_t\\
&\quad-\beta\beta_\delta P_{y,p}\hat{\theta}^{WA}_{p,t}-\beta\beta_\delta 2\kappa^p\varnothing_1P_{\pi,\pi}\hat{P}^{WA}_t
\end{aligned}
$$

$$= -\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} - \frac{P_{y,n^-}}{\beta_\delta}\hat{n}_t$$
$$+P_{y,a}\hat{a}_t^{WA} + P_{y,p}\hat{\theta}_{p,t}^{WA} + 2\kappa^p\varnothing_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big].$$

Finally,

$$\Lambda_{2,t} = -\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} - \frac{P_{y,n^-}}{\beta_\delta}\hat{n}_t$$
$$+P_{y,a}\hat{a}_t^{WA} + P_{y,p}\hat{\theta}_{p,t}^{WA} + 2\kappa^p\varnothing_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big] \quad (189)$$

where $\hat{y}_t^{WA}$, $\hat{n}_t^{WA}$, $\hat{a}_t^{WA}$, $\hat{\theta}_{p,t}^{WA}$, and $\hat{P}_t^{WA}$ are the weighted averages of historical realizations with

$$\hat{y}_t^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{y}_{t-s} \quad (190)$$
$$\hat{n}_t^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{n}_{t-s} \quad (191)$$
$$\hat{a}_t^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{a}_{t-s} \quad (192)$$
$$\hat{\theta}_{p,t}^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{\theta}_{p,t-s} \quad (193)$$
$$\hat{P}_t^{WA} = \sum_{s=0}^{\infty}(\beta_\delta)^s\,\hat{P}_{t-s} \quad (194)$$

or written recursively

$$\hat{y}_t^{WA} = \beta_\delta\hat{y}_{t-1}^{WA} + \hat{y}_t \quad (195)$$
$$\hat{n}_t^{WA} = \beta_\delta\hat{n}_{t-1}^{WA} + \hat{n}_t \quad (196)$$
$$\hat{a}_t^{WA} = \beta_\delta\hat{a}_{t-1}^{WA} + \hat{a}_t \quad (197)$$
$$\hat{\theta}_{p,t}^{WA} = \beta_\delta\hat{\theta}_{p,t-1}^{WA} + \hat{\theta}_{p.t} \quad (198)$$
$$\hat{P}_t^{WA} = \beta_\delta\hat{P}_{t-1}^{WA} + \hat{P}_t. \quad (199)$$

Substituting equation (188) into equation (186) and using the newly defined coefficients,

$$H_t^\Lambda - 2\kappa^p\theta_1 P_{\pi,\pi}\hat{P}_t - 2\kappa^p\beta\theta_2 P_{\pi,\pi}E_t\hat{P}_{t+1} + \left[\frac{1}{\varnothing_1-\phi}\frac{1}{\beta}(\theta_1-\phi) + \frac{1}{\beta}\gamma_1\nu^\Lambda\right]\Lambda_{2,t-1}$$
$$+\left[\frac{1}{\varnothing_1-\phi}(\theta_2+\phi-\theta_1) + \gamma_2\nu^\Lambda\right]\Lambda_{2,t} + \left[\beta\gamma_3\nu^\Lambda - \frac{1}{\varnothing_1-\phi}\beta\theta_2\right]E_t\Lambda_{2,t+1}$$

$$
\begin{aligned}
= \quad & H_t^{\Lambda} - 2\kappa^p\theta_1 P_{\pi,\pi}\hat{P}_t - 2\kappa^p\beta\theta_2 P_{\pi,\pi}E_t\hat{P}_{t+1} + \chi_{-1}^{\Lambda}\left(\frac{1}{\beta_\delta}\Lambda_{2,t} + \beta\left(G_t^{\Lambda} + 2\kappa^p\o_1 P_{\pi,\pi}\hat{P}_t\right)\right) \\
& +\chi_0^{\Lambda}\Lambda_{2,t} + \chi_1^{\Lambda}\left(\beta_\delta\Lambda_{2,t} - \beta\beta_\delta\left(E_t G_{t+1}^{\Lambda} + 2\kappa^p\o_1 P_{\pi,\pi}E_t\hat{P}_{t+1}\right)\right) \\
= \quad & H_t^{\Lambda} + \chi_{-1}^{\Lambda}\beta G_t^{\Lambda} - \chi_1^{\Lambda}\beta\beta_\delta E_t G_{t+1}^{\Lambda} + \left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right)\hat{P}_t \\
& - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)E_t\hat{P}_{t+1} + \left(\frac{\chi_{-1}^{\Lambda}}{\beta_\delta} + \chi_0^{\Lambda} + \chi_1^{\Lambda}\beta_\delta\right)\Lambda_{2,t} \\
= \quad & H_t^{\Lambda} + \chi_{-1}^{\Lambda}\beta G_t^{\Lambda} - \chi_1^{\Lambda}\beta\beta_\delta E_t G_{t+1}^{\Lambda} + \left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right)\hat{P}_t \\
& - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)E_t\hat{P}_{t+1} + \chi^{\Lambda 2}\Lambda_{2,t} \\
= \quad & \left(2P_{n,n} + 2\beta P_{n^-,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n^-} + \chi^{\Lambda 2}\beta\beta_\delta\frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t \\
& + \left(P_{n,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n^-}\right)\hat{n}_{t-1} + \left(\beta P_{n,n^-} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n}\right)E_t\hat{n}_{t+1} \\
& + \left(P_{y,n} + \chi_{-1}^{\Lambda}\beta 2P_{y,y}\right)\hat{y}_t + \left(\beta P_{y,n^-} - \chi_1^{\Lambda}\beta\beta_\delta 2P_{y,y}\right)E_t\hat{y}_{t+1} \\
& + \left(P_{n,a} + \chi_{-1}^{\Lambda}\beta P_{y,a}\right)\hat{a}_t + \left(P_{n,p} + \chi_{-1}^{\Lambda}\beta P_{y,p}\right)\hat{\theta}_{p,t} \\
& -\chi_1^{\Lambda}\beta\beta_\delta P_{y,a}E_t\hat{a}_{t+1} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,p}E_t\hat{\theta}_{p,t+1} \\
& + \left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right)\hat{P}_t - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)E_t\hat{P}_{t+1} \\
& -\chi^{\Lambda 2}\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} + P_{y,a}a_t^{WA} \\
& +P_{y,p}\theta_{p,t}^{WA} + 2\kappa^p\o_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big]. \qquad (200)
\end{aligned}
$$

If the technology shock and the markup shock follow AR(1) process, then

$$
\begin{aligned}
E_t\hat{a}_{t+1} &= \rho_a\hat{a}_t \qquad (201) \\
E_t\hat{\theta}_{p,t+1} &= \rho_p\hat{\theta}_{p,t}. \qquad (202)
\end{aligned}
$$

Together with the definition of price inflation $\pi_t = \hat{P}_t - \hat{P}_{t-1}$, we have

$$
\begin{aligned}
0 = \quad & \left(2P_{n,n} + 2\beta P_{n^-,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n^-} + \chi^{\Lambda 2}\beta\beta_\delta\frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t \\
& + \left(P_{n,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n^-}\right)\hat{n}_{t-1} + \left(\beta P_{n,n^-} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n}\right)E_t\hat{n}_{t+1} \\
& + \left(P_{y,n} + \chi_{-1}^{\Lambda}\beta 2P_{y,y}\right)\hat{y}_t + \left(\beta P_{y,n^-} - \chi_1^{\Lambda}\beta\beta_\delta 2P_{y,y}\right)E_t\hat{y}_{t+1} \\
& + \left(P_{n,a} + \chi_{-1}^{\Lambda}\beta P_{y,a} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,a}\rho_a\right)\hat{a}_t + \left(P_{n,p} + \chi_{-1}^{\Lambda}\beta P_{y,p} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,p}\rho_p\right)\hat{\theta}_{p,t} \\
& + \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p\o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)\right]\hat{P}_t \\
& - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p\o_1 P_{\pi,\pi}\right)E_t\pi_{t+1} \\
& -\chi^{\Lambda 2}\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} + P_{y,a}a_t^{WA} \\
& +P_{y,p}\theta_{p,t}^{WA} + 2\kappa^p\o_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big]
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \left(2P_{n,n} + 2\beta P_{n^-,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n^-} + \chi^{\Lambda 2}\beta\beta_\delta \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t \\
& + \left(P_{n,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n^-}\right)\hat{n}_{t-1} + \left(\beta P_{n,n^-} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n}\right)E_t\hat{n}_{t+1} \\
& + \left(P_{y,n} + \chi_{-1}^{\Lambda}\beta 2P_{y,y}\right)\hat{y}_t + \left(\beta P_{y,n^-} - \chi_1^{\Lambda}\beta\beta_\delta 2P_{y,y}\right)E_t\hat{y}_{t+1} \\
& + \left(P_{n,a} + \chi_{-1}^{\Lambda}\beta P_{y,a} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,a}\rho_a\right)\hat{a}_t + \left(P_{n,p} + \chi_{-1}^{\Lambda}\beta P_{y,p} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,p}\rho_p\right)\hat{\theta}_{p,t} \\
& + \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p \text{\o}_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p \text{\o}_1 P_{\pi,\pi}\right)\right]\pi_t \\
& - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p \text{\o}_1 P_{\pi,\pi}\right)E_t\pi_{t+1} \\
& + \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p \text{\o}_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_\delta 2\kappa^p \text{\o}_1 P_{\pi,\pi}\right)\right]\hat{P}_{t-1} \\
& -\chi^{\Lambda 2}\beta\beta_\delta\Big[2P_{y,y}\hat{y}_t^{WA} + \left(P_{y,n} + \frac{P_{y,n^-}}{\beta_\delta}\right)\hat{n}_t^{WA} + P_{y,a}a_t^{WA} \\
& +P_{y,p}\theta_{p,t}^{WA} + 2\kappa^p \text{\o}_1 P_{\pi,\pi}\hat{P}_t^{WA}\Big]. \qquad (203)
\end{aligned}
$$

Hence, the *optimal targeting rule* is given by,

$$
\begin{aligned}
& \varpi_1\hat{n}_t + \varpi_2\hat{n}_{t-1} + \varpi_3\hat{n}_{t+1} + \varpi_4\hat{y}_t + \varpi_5\hat{y}_{t+1} + \varpi_6\hat{a}_t + \varpi_7\hat{\theta}_{p,t} + \varpi_8\pi_t + \varpi_9\pi_{t+1} \\
& +\varpi_{10}\hat{P}_{t-1} + \varpi_{11}\hat{y}_t^{WA} + \varpi_{12}\hat{n}_t^{WA} + \varpi_{13}\hat{a}_t^{WA} + \varpi_{14}\hat{\theta}_{p,t}^{WA} + \varpi_{15}\hat{P}_t^{WA} = 0 \qquad (204)
\end{aligned}
$$

where we define

$$
\begin{aligned}
\pi_t &= \hat{P}_t - \hat{P}_{t-1} && (205) \\
\hat{y}_t^{WA} &= \beta_\delta\hat{y}_{t-1}^{WA} + \hat{y}_t && (206) \\
\hat{n}_t^{WA} &= \beta_\delta\hat{n}_{t-1}^{WA} + \hat{n}_t && (207) \\
\hat{a}_t^{WA} &= \beta_\delta\hat{a}_{t-1}^{WA} + \hat{a}_t && (208) \\
\hat{\theta}_{p,t}^{WA} &= \beta_\delta\hat{\theta}_{p,t-1}^{WA} + \hat{\theta}_{p.t} && (209) \\
\hat{P}_t^{WA} &= \beta_\delta\hat{P}_{t-1}^{WA} + \hat{P}_t && (210)
\end{aligned}
$$

and

$$
\begin{aligned}
\varpi_1 &= \left(2P_{n,n} + 2\beta P_{n^-,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n^-} + \chi^{\Lambda 2}\beta\beta_\delta\frac{P_{y,n^-}}{\beta_\delta}\right) \\
\varpi_2 &= \left(P_{n,n^-} + \chi_{-1}^{\Lambda}\beta P_{y,n^-}\right) \\
\varpi_3 &= \left(\beta P_{n,n^-} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,n}\right) \\
\varpi_4 &= \left(P_{y,n} + \chi_{-1}^{\Lambda}\beta 2P_{y,y}\right) \\
\varpi_5 &= \left(\beta P_{y,n^-} - \chi_1^{\Lambda}\beta\beta_\delta 2P_{y,y}\right) \\
\varpi_6 &= \left(P_{n,a} + \chi_{-1}^{\Lambda}\beta P_{y,a} - \chi_1^{\Lambda}\beta\beta_\delta P_{y,a}\rho_a\right)
\end{aligned}
$$

$$
\begin{aligned}
\varpi_7 &= \left(P_{n,p} + \chi_{-1}^{\Lambda}\beta P_{y,p} - \chi_1^{\Lambda}\beta\beta_{\delta}P_{y,p}\rho_p\right) \\
\varpi_8 &= \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p \o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_{\delta}2\kappa^p\o_1 P_{\pi,\pi}\right)\right] \\
\varpi_9 &= -\left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_{\delta}2\kappa^p\o_1 P_{\pi,\pi}\right) \\
\varpi_{10} &= \left[\left(\chi_{-1}^{\Lambda}\beta 2\kappa^p \o_1 P_{\pi,\pi} - 2\kappa^p\theta_1 P_{\pi,\pi}\right) - \left(2\kappa^p\beta\theta_2 P_{\pi,\pi} + \chi_1^{\Lambda}\beta\beta_{\delta}2\kappa^p\o_1 P_{\pi,\pi}\right)\right] \\
\varpi_{11} &= -\chi^{\Lambda 2}\beta\beta_{\delta}2P_{y,y} \\
\varpi_{12} &= -\chi^{\Lambda 2}\beta\beta_{\delta}\left(P_{y,n} + \frac{P_{y,n^-}}{\beta_{\delta}}\right) \\
\varpi_{13} &= -\chi^{\Lambda 2}\beta\beta_{\delta}P_{y,a} \\
\varpi_{14} &= -\chi^{\Lambda 2}\beta\beta_{\delta}P_{y,p} \\
\varpi_{15} &= -\chi^{\Lambda 2}\beta\beta_{\delta}2\kappa^p\o_1 P_{\pi,\pi}
\end{aligned}
$$

with additional parameters being defined as

$$
\begin{aligned}
\o_1 &= \phi + \frac{\varpi^{y_{ss}}\sigma}{1-\kappa^c} \\
\kappa^p &= \frac{(1-\beta\xi^p)(1-\xi^p)}{\xi^p} \\
\nu^{\Lambda} &= -\frac{\frac{1}{\varpi^{y_{ss}}}\frac{1-\kappa^c}{\sigma}}{\frac{(1-\rho)\beta}{\mu}\left(1-\xi q_{ss}\theta_{ss}\right)\left(\frac{\kappa^v}{q_{ss}} + \bar{\kappa}\right)} \\
\chi_{-1}^{\Lambda} &= \left[\frac{1}{\o_1 - \phi}\frac{1}{\beta}(\theta_1 - \phi) + \frac{1}{\beta}\gamma_1\nu^{\Lambda}\right] \\
\chi_0^{\Lambda} &= \left[\frac{1}{\o_1 - \phi}\left(\theta_2 + \phi - \theta_1\right) + \gamma_2\nu^{\Lambda}\right] \\
\chi_1^{\Lambda} &= \left[\beta\gamma_3\nu^{\Lambda} - \frac{1}{\o_1 - \phi}\beta\theta_2\right] \\
\beta_{\delta} &= \frac{1}{\beta\left[1 - (1+\o_1)\,\nu^{\Lambda}\right]} \\
\chi^{\Lambda 2} &= \left(\frac{\chi_{-1}^{\Lambda}}{\beta_{\delta}} + \chi_0^{\Lambda} + \chi_1^{\Lambda}\beta_{\delta}\right).
\end{aligned}
$$

D Optimal targeting rule for NK model with sticky nominal wages

To find the optimal targeting rule in the sticky wage model, we follow Giannoni and Woodford (2003). The linear quadratic problem can be shown to be

$$
\min_{\{\pi_t,\pi_t^w,x_t,i_t,\hat{w}_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t\left\{\frac{\sigma+\phi}{2}x_t^2 + \frac{1+\theta^p}{2\theta^p\kappa^p}\pi_t^2 + \frac{1+\theta^w}{2\theta^w\kappa^w}(\pi_t^w)^2\right\}
$$

$$
\begin{aligned}
s.t. \quad & x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \hat{r}_t^*\right) && (\Lambda_{1,t}) \\
& \pi_t = \beta E_t \pi_{t+1} + \kappa^p \left(\hat{w}_t - \hat{a}_t\right) + \hat{\theta}_{p,t} && (\Lambda_{2,t}) \\
& \pi_t^w = \beta E_t \pi_{t+1}^w + \kappa^w \left(\sigma + \phi\right) x_t - \kappa^w \left(\hat{w}_t - \hat{a}_t\right) && (\Lambda_{3,t}) \\
& \hat{w}_t = \hat{w}_{t-1} + \pi_t^w - \pi_t && (\Lambda_{4,t}) \qquad (211)
\end{aligned}
$$

where

$$\theta^p = \lambda^p - 1 \tag{212}$$

$$\theta^w = \lambda^w - 1 \tag{213}$$

$$\kappa^p = \frac{(1-\xi^p)(1-\xi^p\beta)}{\xi^p} \tag{214}$$

$$\kappa^w = \frac{(1-\xi^w)(1-\xi^w\beta)}{\xi^w}. \tag{215}$$

The first order conditions associated with the policymaker's preferences are

$$(\pi_t): \quad \frac{1+\theta^p}{\theta^p\kappa^p}\pi_t + \frac{\beta^{-1}}{\sigma}\Lambda_{1,t-1} + \Lambda_{2,t-1} - \Lambda_{2,t} - \Lambda_{4,t} = 0 \tag{216}$$

$$(\pi_t^w): \quad \frac{1+\theta^w}{\theta^w\kappa^w}(\pi_t^w) + \Lambda_{3,t-1} - \Lambda_{3,t} + \Lambda_{4,t} = 0 \tag{217}$$

$$(x_t): \quad (\sigma+\phi)\, x_t + \beta^{-1}\Lambda_{1,t-1} - \Lambda_{1,t} + \kappa^w(\sigma+\phi)\,\Lambda_{3,t} = 0 \tag{218}$$

$$(i_t): \quad \frac{1}{\sigma}\Lambda_{1,t} = 0 \tag{219}$$

$$(w_t): \quad \kappa^p\Lambda_{2,t} - \kappa^w\Lambda_{3,t} + \beta\Lambda_{4,t+1} - \Lambda_{4,t} = 0. \tag{220}$$

From equation (219), we obtain

$$\Lambda_{1,t} = 0 \qquad \forall t. \tag{221}$$

Accordingly, the optimality conditions simplify to

$$(\pi_t): \quad \frac{1+\theta^p}{\theta^p\kappa^p}\pi_t + \Lambda_{2,t-1} - \Lambda_{2,t} - \Lambda_{4,t} = 0 \tag{222}$$

$$(\pi_t^w): \quad \frac{1+\theta^w}{\theta^w\kappa^w}(\pi_t^w) + \Lambda_{3,t-1} - \Lambda_{3,t} + \Lambda_{4,t} = 0 \tag{223}$$

$$(x_t): \quad (\sigma+\phi)\, x_t + \kappa^w(\sigma+\phi)\,\Lambda_{3,t} = 0 \tag{224}$$

$$(w_t): \quad \kappa^p\Lambda_{2,t} - \kappa^w\Lambda_{3,t} + \beta\Lambda_{4,t+1} - \Lambda_{4,t} = 0. \tag{225}$$

From equation (224),

$$\Lambda_{3,t} = -\frac{1}{\kappa^w} x_t. \tag{226}$$

Then substituting $\Lambda_{3,t}$ into equation (223), we get an expression of $\Lambda_{4,t}$

$$\Lambda_{4,t} = -\frac{1+\theta^w}{\theta^w \kappa^w}(\pi_t^w) - \frac{1}{\kappa^w} x_t + \frac{1}{\kappa^w} x_{t-1}. \tag{227}$$

Plugging the expressions for $\Lambda_{3,t}$ and $\Lambda_{4,t}$ into equation (225) delivers $\Lambda_{2,t}$

$$\begin{aligned} \Lambda_{2,t} \quad &= \beta \frac{1+\theta^w}{\theta^w \kappa^w \kappa^p}(\pi_{t+1}^w) - \frac{1+\theta^w}{\theta^w \kappa^w \kappa^p}(\pi_t^w) + \frac{\beta}{\kappa^w \kappa^p} x_{t+1} \\ &+ \frac{1}{\kappa^w \kappa^p} x_{t-1} - \left(\frac{\beta}{\kappa^w \kappa^p} + \frac{1}{\kappa^w \kappa^p} + \frac{1}{\kappa^p} \right) x_t. \end{aligned} \tag{228}$$

After substituting the expressions for $\Lambda_{2,t}$ and $\Lambda_{4,t}$ into equation (222) and using the definition of the output gap ($x_t = \hat{y}_t - \frac{1+\phi}{\sigma+\phi}\hat{a}_t$), the *optimal targeting rule* for sticky wage model is given by

$$\begin{aligned} -\chi_1 \pi_t \quad = \quad & \chi_2 \left(\pi_{t+1}^w - \pi_t^w \right) + \chi_3 \pi_t^w + \chi_4 \left(\pi_t^w - \pi_{t-1}^w \right) + \chi_5 \left[(\hat{y}_{t+1} - \hat{y}_t) - \frac{1+\phi}{\sigma+\phi} (\hat{a}_{t+1} - \hat{a}_t) \right] \\ & + \chi_6 \left[(\hat{y}_t - \hat{y}_{t-1}) - \frac{1+\phi}{\sigma+\phi} (\hat{a}_t - \hat{a}_{t-1}) \right] + \chi_7 \left[(\hat{y}_{t-1} - \hat{y}_{t-2}) - \frac{1+\phi}{\sigma+\phi} (\hat{a}_{t-1} - \hat{a}_{t-2}) \right] \end{aligned} \tag{229}$$

where

$$\begin{aligned} \chi_1 &= \frac{1+\theta^p}{\theta^p} \frac{1}{\kappa^p} \\ \chi_2 &= -\beta \frac{1+\theta^w}{\theta^w} \frac{1}{\kappa^p \kappa^w} \\ \chi_3 &= \frac{1+\theta^w}{\theta^w} \frac{1}{\kappa^w} \\ \chi_4 &= \frac{1+\theta^w}{\theta^w} \frac{1}{\kappa^p \kappa^w} \\ \chi_5 &= -\frac{\beta}{\kappa^p \kappa^w} \\ \chi_6 &= \left(\frac{1}{\kappa^p \kappa^w} + \frac{\beta}{\kappa^p \kappa^w} + \frac{1}{\kappa^p} + \frac{1}{\kappa^w} \right) \\ \chi_7 &= -\frac{1}{\kappa^p \kappa^w}. \end{aligned}$$